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Multiple Information Sources and Equilibrium in Financial Markets

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by

Elyashiv David Wiedman

2020

ABSTRACT OF THE DISSERTATION

Multiple Information Sources

and Equilibrium in Financial Markets

by

Elyashiv David Wiedman

Doctor of Philosophy in Management

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Professor Brett Michael Trueman, Chair

The purpose of the dissertation is to examine the interaction among multiple information sources in financial markets such as among analysts and firm managers and the effect of these interactions on the information available in financial markets.

In the first chapter, analysts of heterogeneous abilities may choose which firm to cover, firms with or without prior coverage, and then issue earnings forecasts. The information an analyst observes depends on his ability and may be correlated with that of other analysts who cover the same firm. It is shown that analysts with career concerns, i.e., who aim to appear well informed, may differ in their coverage decisions. Thus, analyst initiations of coverage provide information about analysts' abilities. Also, depending on the correlation of analysts' information and the prevalence of competent analysts, analysts' reputations are not necessarily monotonic in forecast accuracy, in contrast to the prevalent assumption in the empirical literature. Further, incompetent analysts bias their forecast if their information is not sufficiently precise. It is shown that forecasting bias is more likely when a few analysts cover the same firm than when

an analyst initiates exclusive firm coverage.

In the second chapter, the interaction between analysts and firms' managers is examined. A firm's manager may be regarding future demand and may disclose it at his discretion. An analyst who covers the firm's industry observes relevant information and issues a (possibly biased) forecast. Whether the analyst issues a biased or unbiased forecast is unknown to investors, who price the firm based on the available information. It is shown that investors' beliefs about the manager's information endowment and the analyst's forecasting objective are endogenously intertwined. Thus, in addition to the direct role of the analyst's forecast in providing information, the forecast has an indirect effect by influencing investors' beliefs regarding the manager's information endowment. If the analyst's forecast comes after the manager's possible disclosure, it is shown that analyst coverage crowds out the manager's disclosure compared to the case without coverage. If the analyst's forecast precedes the manager's disclosure, the manager may disclose his information even when the analyst issues a positively biased forecast.

The dissertation of Elyashiv David Wiedman is approved.

John S. Hughes

Judson Andrew Caskey

Henry L. Friedman

Beatrice Michaeli

Brett Michael Trueman, Committee Chair

University of California, Los Angeles

2020

To Nava, my love.

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Vita

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CHAPTER 1

Analyst Initiations of Coverage

1 Introduction

Sell-side analysts play a central role in financial markets through the information they provide to market participants. However, the information analysts provide varies in quality because analysts vary in abilities; see, for example, Desai, Liang, and Singh (2000) and Fang and Yasuda (2014). Thus, it is not surprising that investors, employers, and academic researchers have invested great efforts to identifying the underlying abilities of analysts. In turn, analysts aim to develop a reputation as being well informed. Earnings forecasts are a piece of valuable information that produced by analysts, and thus, are considered a useful source in determining analysts' reputations, see, for example, Jackson (2005). In addition to earnings forecasts, information about the underlying competency of an analyst may be revealed from other actions that analysts take, such as the choice of which firms they cover.[1] Surprisingly, the information that can be inferred from coverage decisions has been overlooked by previous literature.[2] This paper examines the relationship between analysts' reputations, their coverage decisions, and earnings forecasts.

[1]It is not immediate that analysts have discretion over the firm they cover. In discussions with experts in the field, it was suggested that at least some analysts have such a choice. Lee and So (2017) provide empirical evidence that at least some analysts choose the firms they cover. The model allows for some analysts to choose the firms they cover, while others are endogenously assigned to cover certain firms.

[2]One an exception is Lee and So (2017), who study the information arising from excess coverage of analysts. They focus on the information that can be learned about the firm's future profitability and not about the information that arises regarding analysts' abilities.

Whether earnings forecast accuracy affects analysts' abilities to influence market prices and whether forecast accuracy influences analysts' reputations are the focus of many empirical studies. For example, Jackson (2005) shows that accurate forecasts generate a higher reputation and that highly reputed analysts generate more trade for their brokerage firms. Brown (2001) demonstrates that past forecast accuracy is useful in predicting an analyst's future forecast accuracy. Clement (1999), Makhil et al. (1999), Hong and Kubik (2003) and Jackson (2005), among many others, analyze how forecast accuracy affects the development of an analyst career, e.g., the level of prestige of the brokerage house wherein he is employed or the probability of work termination.[3] While some papers found evidence for forecast accuracy positively affecting an analyst's career outcome, other studies did not find such a result. Forecast accuracy is the focus in vast literature because forecasts are observed by investors and researchers and can be compared to firms' earnings when announced. However, another observable action, namely, analysts' choice of the firms they cover, has been overlooked by most of the literature, in particular the theoretical research. Lee and So (2017) examine the information that can be learned about a firm's future earnings from analysts' coverage decisions. In this paper, I raise the question as to whether analysts of heterogeneous abilities choose to follow firms with or without prior analyst coverage based on their competency, and in turn, whether analysts' coverage decision should be used when determining analysts' reputations as being well informed.

I study a model where a new analyst, i.e., an analyst with no prior reputation, may follow only one of the two firms in the market. Firms are identical in their characteristics, and the only difference arises from analysts' coverage; an established analyst

[3] In a recent survey of analysts by Brown at el. (2015), only 25% of the surveyed analysts indicated that forecast accuracy is highly relevant to their compensation. However, in the same survey, more than half of the analysts noted that they exclude components of GAAP earnings because of their desire to improve their earnings forecast accuracy. Therefore, it is clear that forecast accuracy is essential for analysts.

follows one of the firms. The new analyst can choose whether to follow the same firm as the established analyst or to cover a firm without prior coverage exclusively. Analysts observe noisy signals about the future earnings of the firm they follow. The noisiness of the signals depends on analysts' types, i.e., their competency, which is only privately known to the analysts. Investors, who are the clients of the analysts, revise their beliefs about analysts' competency based on the information they observe; that is, analysts' coverage decisions and their forecast accuracy. In contrast to prior literature, I assume that the signals of competent analysts are conditionally correlated. Thus, if the new analyst follows the same firm as the established one, then the signals they observe may be correlated. Therefore, an analyst's reputation, i.e., investors' beliefs regarding his competency, may also depend on his peer analyst forecast. In Appendix B, I consider the opposite case, where the signals of incompetent analysts are conditionally correlated; The results remain qualitatively the same.

I start the analysis in Section III, focusing on the relationship between forecast accuracy and (ex-post) reputation while muting the analysts' initiation of coverage decisions, i.e., assuming that initiation of coverage is exogenously determined. In this case, the only information available to investors is the firms' earnings, the new analyst's forecast, and the forecast that the established analyst issues for the firm he covers.[4] I show that if the new analyst follows the firm with no prior coverage, his reputation increases in his (absolute) forecast accuracy. This result is in the same spirit as many empirical papers, e.g., Clement (1999) and Jackson (2005), where proxies used for analysts' reputations are positively correlated with their measures of forecast accuracy.

When the new analyst follows the same firm as the established one, investors' abilities to update their beliefs about his competency improve because of the additional

[4]In case that both analysts cover the same firm, I assume that they issue their forecasts simultaneously. Hence, an analyst does not learn any information about his peer's forecast, and so, no observational herding arises.

information arising from the established analyst's forecast. I show, perhaps surprisingly, that an analyst's reputation, i.e., investors' beliefs about his competency, is not monotonic in his forecast accuracy if the correlation of information for competent analysts is sufficiently high and the established analyst is likely to be competent.[5] The intuition for the lack of monotonicity is the following: competent analysts are likely to observe similar information, and thus analysts might be better off when both issue inaccurate forecasts than when one issues a more accurate forecast while his peer issues an inaccurate one. Correlation between competent analysts' information may arise because they have access to the same information before issuing their forecast or because better-qualified analysts use similar models to determine the firm's future earnings. In this paper, the signals' conditional correlation plays a vital role, in contrast to the widespread assumption in prior literature of conditionally independent signals, for example, Trueman (1994). Banerjee (2020) also considers a model where analysts' signals are conditionally correlated. He shows that when analysts face greater competition, it may dampen the aggregate information that investors receive. While this study and Banrjee (2020) both assume that signals are conditionally correlated, I assume that analysts are of heterogeneous competency and aim to maximize their reputation while in Banerjee's paper, homogeneous analysts care about their absolute and relative accuracy.

In Section IV, I endogenize analysts' initiation of coverage decisions. Whether analysts have discretion over the firms they cover is an empirical question. In discussions with experts in the this field, it was suggested that at least some analysts possess such discretion. Lee and So (2017) provide empirical evidence that initiation of coverage may depend on an analyst's private information. Accordingly, I assume that an analyst may choose the firm he covers with some probability, q. I show that, depending on

[5] I determine exact conditions for this non-monotonicity to hold.

the probability that an analyst has coverage discretion, the equilibrium is pooling or a separating one. When the new analyst has full discretion over the firm he covers, he decides to follow the same firm as the established analyst, regardless of his type. However, if analysts have coverage discretion with only some probability (less than one), initiation of coverage depends on the analysts' types. Specifically, a competent analyst chooses to cover the same firm as the established one, while an incompetent analyst elects to follow the firm without prior coverage, at least with positive probability.[6] As a result, in equilibrium, an analyst who initiates coverage for the same firm as the established one is more likely to be competent than an analyst who covers the other firm exclusively. In other words, an analyst's reputation depends on the firm he covers, even before considering his forecast accuracy.

It is well established in the literature that analysts sometimes bias their forecasts, see, for example, Trueman (1994) and Michaely and Womak (1999). In Section V, I consider the possibility that analysts bias their earnings forecasts. Like Trueman (1994) and Aharoni et al. (2017), I show that incompetent analysts underweight their private information and thus bias their forecast toward the firm's prior expected earnings. I further demonstrate that, while incompetent analysts bias their forecasts regardless of the firm they cover, forecasting bias is less likely to occur when the new analyst follows a firm exclusively. The reason for a more significant bias when covering the same firm as the established analyst is the correlation of competent analysts' information. The new analyst is better off when his forecast is similar to that of the established one. Therefore, an incompetent analyst has a greater incentive to bias his forecast since he expects the established analyst's forecast to be the same as the expected earnings.

[6] I show that a pure strategy equilibrium does not always exist. In such a case, a mixed strategy equilibrium exists where incompetent analysts mix between covering each of the firms.

2 Model

There are two firms in the market, A and B. The firms are ex-ante identical; differences arise only due to variation between the analysts who follow each firm. Specifically, I assume that an established analyst, e, follows firm A, while there is no prior coverage of firm B. A new analyst, n, who enters the market may follow either firm A or firm B. I start by describing the earnings of a single firm and the information of a single analyst who covers that firm.[7] Therefore, I postpone the use of the firms' related indices to the next section.

2.1 The firm's earnings and the analyst's information

The earnings of a firm, π, are uncertain and can take four possible values, $\pi \in \{\pi^{vh}, \pi^{mh}, \pi^{ml}, \pi^{vl}\}$; positive earnings may be very high (π^{vh}) or moderately high (π^{mh}), while negative earnings take moderately low and very low values, (π^{ml}, π^{vl}).[8] I assume that earnings are symmetric around zero so $\pi^{vh} = -\pi^{vl}$ and $\pi^{mh} = -\pi^{ml}$. The prior probability of earnings is $Pr(\pi^{vh}) = Pr(\pi^{vl}) = \frac{1-\alpha}{2}$ and $Pr(\pi^{mh}) = Pr(\pi^{ml}) = \frac{\alpha}{2}$. I assume that $\frac{1}{2} \leq \alpha$; that is, moderate earnings are (weakly) more likely than extreme earnings, either positive or negative.[9]

An analyst who covers the firm privately observes (possibly noisy) information about the firm's earnings and issues an earnings forecast. Specifically, the analyst observes

[7] The basic structure of the model, i.e., the firm's earnings and analysts' signals, is similar to that of Trueman (1994). However, I diverge from his model by assuming that analysts' information is conditionally correlated and that analysts may choose the firm they cover.

[8] The model can be generalized to the case of a continuous state space with a uniform prior. The results in Sections 3 and 4 remain qualitatively the same; the uniform prior yields honest forecasts which derive the same results. However, the results of Section 5 regarding biased forecasts cannot be extended to a model with a continuous state space because there is no equilibrium where analysts bias their forecasts towards a specific state (or forecast), see Ottaviani and Sorensen (2006).

[9] The assumption of symmetry around zero is for convenience only and does not affect the results. The assumption that moderate earnings are more likely than extreme earnings affects analysts forecasting bias, which I discuss in Section 5. If we assume that extreme earnings are more likely than moderate earnings, then a less competent analyst would bias his forecast toward extreme earnings.

a signal, $s \in \{s^{vh}, s^{mh}, s^{ml}, s^{vl}\}$, about the firm's earnings. I assume that the analyst learns for sure whether the firm's earnings is positive (π^{vh}, π^{mh}), or negative (π^{ml}, π^{vl}), but the signal is noisy with regards to the magnitude of the earnings, e.g., very high or high. That is,

$$
\begin{aligned}
&Pr(s^{jk}|\pi^{jk}) = p && for \quad j \in \{m, v\} \; and \; k \in \{h, l\} \\
&Pr(s^{jk}|\pi^{j'k}) = 1 - p && for \quad j, j' \in \{m, v\}, j \neq j' \; and \; k \in \{h, l\} \\
&Pr(s^{jk}|\pi^{j'k'}) = 0 && for \quad j, j' \in \{m, v\} \; and \; k, k' \in \{h, l\}, k \neq k'
\end{aligned}
$$

After observing his signal, an analyst updates his expectation about the firm's earnings. While the analyst learns that the firm's earnings is positive or negative, there is still some uncertainty about the magnitude of the earnings. The updated earnings probability distribution after observing a signal is then

$$
\begin{aligned}
Pr(\pi^{mh}|s^{mh}) = Pr(\pi^{ml}|s^{ml}) &= \frac{p\alpha}{p\alpha + (1-p)(1-\alpha)} \\
Pr(\pi^{vh}|s^{mh}) = Pr(\pi^{vl}|s^{ml}) &= \frac{(1-p)(1-\alpha)}{p\alpha + (1-p)(1-\alpha)} \\
Pr(\pi^{vh}|s^{vh}) = Pr(\pi^{vl}|s^{vl}) &= \frac{p(1-\alpha)}{p(1-\alpha) + (1-p)\alpha} \\
Pr(\pi^{mh}|s^{vh}) = Pr(\pi^{ml}|s^{vl}) &= \frac{(1-p)\alpha}{p(1-\alpha) + (1-p)\alpha}
\end{aligned}
\tag{1}
$$

And so, the expected firm's earnings after observing a signal s^{jk}, $k \in \{h, l\}$, is given by

$$
\begin{aligned}
E[\pi|s^{vk}] &= \frac{p(1-\alpha)\pi^{vk} + (1-p)\alpha\pi^{mk}}{p(1-\alpha) + (1-p)\alpha} \\
E[\pi|s^{mk}] &= \frac{(1-p)(1-\alpha)\pi^{vk} + p\alpha\pi^{mk}}{p\alpha + (1-p)(1-\alpha)}
\end{aligned}
\tag{2}
$$

Because of the symmetry of positive and negative earnings, in the following subsections I forego the sign index $k = h, l$, while using only the magnitude index $j = m, v$.

2.2 Analysts' abilities and correlation of information

Analysts differ in their ability to forecast firms' earnings accurately, see Desai et al. (2000) and Fang and Yasuda (2014); a more competent analyst is expected to provide a more accurate forecast than a less competent one. These differences may arise, for example, due to analysts' abilities to acquire firm-relevant and industry-relevant information, or their ability to analyze the information they receive. Each analyst is either competent, which I denote by *good*, or incompetent, denoted by *bad*, with the prior probability $Pr(\text{analyst is } good) = \theta$. The accuracy of his signal reflects the ability of an analyst. That is, a competent analyst observes a better (in a Blackwell sense) signal than an incompetent one does. Accordingly, I assume that p equals g for competent analysts, and p equals b for incompetent ones, where $\frac{1}{2} \leq b < g \leq 1$.

Differences in analysts' abilities also affect the correlation of their information. For example, two competent analysts may have some industry-specific knowledge that influences, in a similar way, the earnings-related information they possess. Accordingly, I assume that the signals of competent analysts are conditionally correlated.[10] Formally, assuming that two analysts, say 1 and 2, are competent and observe the signals s_1^j and $s_2^{j'}$, $j, j' \in \{m, v\}$, then

$$\begin{aligned} &Pr(s_1^j = s_2^j | \pi^j) = \rho g + (1-\rho) g^2 \\ &Pr(s_1^j, s_2^j | \pi^{j'}) = \rho(1-g) + (1-\rho)(1-g)^2 \qquad j \neq j' \\ &Pr(s_1^j, s_2^{j'} | \pi^j) = Pr(s_1^{j'}, s_2^j | \pi^j) = (1-\rho) g (1-g) \quad j \neq j' \end{aligned} \tag{3}$$

The parameter ρ reflects the (conditional) correlation between competent analysts' signals, where $\rho = 1$ represents full (conditional) correlation, while $\rho = 0$ reflects no correlation. As I discuss in length below, the (conditional) correlation of competent

[10] I assume that the signal of an incompetent analyst is conditionally independent of that of any other analyst, competent or incompetent.

analysts' information plays a key role in this paper.

An analyst's ability is an important aspect for investors who use the information he provides. However, although an analyst is conscious of his own ability, other market participants do not know his ability but rather form beliefs about the analyst's ability. I assume that analysts are fully aware of their own type. However, analysts' abilities are privately known, and all the other parties, e.g., investors, initially use the prior distribution of analysts' types, Pr(an analyst is *good*) $= \theta$, and then update their beliefs about an analyst's competency using the information they observe. Denote by $\tilde{\theta}_i(Information) = Pr(\text{analyst } i \text{ is } good|Information)$ the probability that investors attribute to an analyst being competent given their information.

The initial reputation of the new analyst, θ_n, and that of the established analyst, θ_e, are assumed to be independent, and need not be the same. The reputation of the established analyst may reflect investors' beliefs about his competency based on the information they possess from past activities. The new analyst's initial reputation may reflect the prevalence of competent analysts in the brokerage industry.

Past literature demonstrates that investors' beliefs about an analyst's ability influence his compensation; see, for example, Jackson (2005). As in Trueman (1994), Ottaviani and Sorensen (2006), among many others, I assume that analysts aim to appear as being well informed, i.e., to maximize investors' beliefs that they are competent, $\tilde{\theta}_i(Information)$.

2.3 Analysts' forecasts and reputations

After observing his signal and updating the expectation about the firm's earnings, analyst i issues his forecast for the earnings of the firm he follows, f_i.[11] Investors use the

[11]Note that the analyst's forecast, f_i, is not necessarily reflecting his expectation for the firm's earnings or the information he observed since the analyst may bias his earnings forecast.

analyst's earnings forecast to make better investment decisions, and then, when the firm's earnings are announced, investors also use this forecast to update their beliefs about the analyst's competency; that is, $\tilde{\theta}_i(f_i, f_{-i}, \pi)$, where f_{-i} is the forecast of another analyst who follows the same firm, if exists, and π is the realized earnings of the firm.

Note that competent and incompetent analysts must use the same set of possible forecasts in their reports; otherwise, investors will identify an analyst's competency from the forecast he issues even before the firm's earnings announcement takes place. In other words, it is impossible to differentiate between the different types of analysts based solely on the forecasts they issue, without any reference to other information.

REMARK **1.** *Let $\mathcal{F}$ be the set of forecasts used by competent analysts, then, in equilibrium, an incompetent analyst issues only forecast in $\mathcal{F}$.*

Nonetheless, as I show below, investors use the forecasts that analysts issue together with the announced earnings, i.e., forecasts accuracy, to determine analysts' reputations.

Since analysts observe only two possible signals conditional on the firm's earnings and analysts learn for sure whether the firm's earnings is positive or negative, w.l.o.g. I restrict attention to the forecasts set $\mathcal{F} = \{f^v, f^m\}$.

3 Reputation with exogenous firms coverage

Before examining the initiation-of-coverage decision, I first analyze analysts' forecasts and reputations in case the new analyst, n, cannot choose the firm he covers, but rather is exogenously assigned to cover a firm. I relax this assumption in the next section. I continue this section without a firm index but specify whether the firm is followed by one analyst or more. The timeline is as shown in Figure 1.

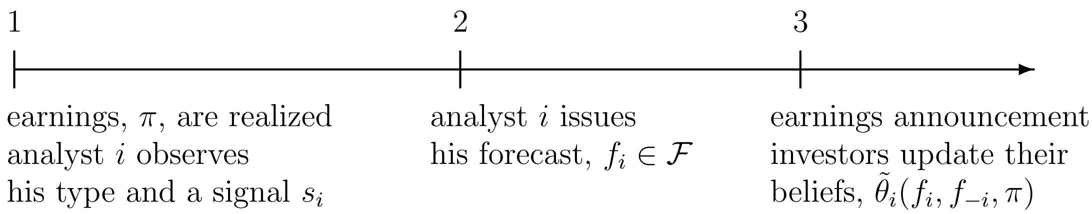

Figure 1: Earnings forecasts and reputation

After observing his signal, in equilibrium, the analyst chooses the probability with which he issues each of the possible forecasts based on his expectation of the firm's earnings, and, if it exists, the expected forecast of the other analyst who covers the same firm. After earnings are announced, investors update their beliefs about the analyst's competency, i.e., determine his reputation. The equilibrium concept is Perfect Bayesian Equilibrium (henceforth 'equilibrium').

Definition **1.** *(equilibrium)*

Let Δ be a probability measure over $\mathcal{F}$.

An equilibrium is a set of forecasting strategies $\sigma_i, \sigma_{-i} \in \Delta$ and investors beliefs $\tilde{\theta}_i(f_i, f_{-i}, \pi)$ such that:

(i) for any signal s_i, $\sigma_i(s_i)$ solves

$$\sigma_i \in argmax \;\; \mathbb{E}_{\pi, f_{-i}}[\tilde{\theta}_i(f_i(\sigma_i), f_{-i}(\sigma_{-i}), \pi)|s_i]$$

where $Pr(\pi|s_i)$ and $Pr(s_{-i}|s_i)$ are consistent with Bayes' rule.

(ii) Investors update their beliefs, $\tilde{\theta}_i(f_i(\sigma_i), f_{-i}(\sigma_{-i}), \pi)$, using Bayes' rule, whenever possible.

(iii) Off-equilibrium beliefs are such that any forecast $f_i' \notin \mathcal{F}$ results in the belief

$\tilde{\theta}_i(f_i', \cdot) = 0.$

I focus on the most informative equilibrium, the equilibrium where investors receive the most information possible about the firm's earnings, compared to any other equilibrium.[12]

To shed light on differences in analysts' reputations that arise when covering each of the two firms, and differences in coverage decisions, I restrict attention, in the next two sections, to cases where analysts do not bias their forecasts. That is, cases where, in equilibrium, analysts' forecasting strategies are one-to-one mapping from signals to forecasts.[13]

ASSUMPTION **1.** $\alpha = \frac{1}{2}$

Assumption 1 ensures that, in equilibrium, analysts issue forecasts according to their signals without any bias; I relax Assumption 1 in Section 5 to analyze the possibility of forecast bias. It is important to note that the assumption imposes restrictions on the earnings distribution rather than on the possible forecasts.

In equilibrium, analysts' forecasts reveal the information they observe, as suggested in the following lemma.

Lemma 1 (Honest forecasting equilibrium)**.**
Assume $\alpha = \frac{1}{2}$. In equilibrium analysts issue forecasts according their signals, i.e.,

$$f_i(s_i) = \begin{cases} f_i^v & if \quad s_i = s_i^v \\ f_i^m & if \quad s_i = s_i^m \end{cases}$$

Proof: See Appendix.

[12]That is, given the equilibrium strategy σ_i, in case that the analyst observes the signal s_i and issues the forecast f_i then $Pr(s_i|f_i(\sigma_i))$ is maximized compared to any other equilibrium.

[13]Note that I do not restrict analysts to issue forecasts according to their signals but rather focus on the case where the prior distribution of the firm's earnings is such that, in equilibrium, analysts issue forecasts according to their signals.

When the possible states, i.e., firm's earnings, are equally likely, analysts issue forecasts according to their signals. This honest equilibrium arises because analysts' signals are informative about the firm's earnings and, if relevant, about the other analyst's signal. By issuing a forecast according to his signal, an analyst maximizes the probability that his forecast matches the earnings realization and the other analyst's forecast. Note that this result is not necessarily true when $\alpha \neq \frac{1}{2}$, i.e., some earnings are a priori more likely than others. I relax Assumption 1 in Section 5 to discuss this possibility.

Investors observe the forecasts that analysts provide. In equilibrium, analysts issue forecasts according to their signals, and so, investors learn the observed signals. Nonetheless, investors are uncertain about the signal accuracy, i.e., the analyst's competency, thus they form beliefs about each analyst's competency. After the earnings announcement, investors update these beliefs using the information they possess. Prior literature focuses on analysts' forecast accuracy, or forecast error, as a measure for market beliefs about analysts' competency. There is mixed evidence regarding the predictive power of an analyst's past forecast accuracy on future performance. While reputation is monotonic in forecast accuracy for an analyst who is the only one to cover a firm, the next proposition suggests it is not necessarily the case when a few analysts cover the same firm.

Proposition 1 (Reputation and forecast accuracy)**.**

(i) When a single analyst covers a firm, his reputation, $\tilde{\theta}_i(f_i, \pi)$, is monotonic in his forecast accuracy.

(ii) When two analysts cover the same firm, the new analyst's reputation, $\tilde{\theta}_n(f_n, f_e, \pi)$, is not monotonic in his forecast accuracy if the established analyst is sufficiently likely to be competent, and the correlation between competent analysts is sufficiently high. That is, if $\overline{\theta} < \theta_e$ and $\overline{\rho} < \rho$, where $\overline{\theta} = \frac{(g-b)(1-b)}{g(1-b)-b(g-b)}$ and

$\overline{\rho} = \frac{(g-b)(1-b-\theta_e(g-b))}{\theta_e g(1-g)}$.

Otherwise, the new analyst's reputation, $\tilde{\theta}_n(\cdot)$, is monotonic in his forecast accuracy.

Proof: See Appendix.

When only one analyst covers a firm, his reputation is monotonic in his forecast accuracy. The intuition for this monotonicity is simple; a more competent analyst observes more accurate information than a less competent one. Thus, an accurate forecast is more likely to be issued by a competent analyst, which results in a better reputation. However, when there is more than one analyst who covers a firm, information about an analyst's competency arises from his forecast accuracy and his peer's forecast. Since competent analysts are assumed to observe (conditionally) correlated information, one analyst's forecast may be informative about the competency of another analyst. Proposition 1 suggests that when the established analyst is sufficiently likely to be competent, and the correlation between competent analysts is sufficiently high, then the reputation of the new analyst is not monotonic in his forecast accuracy. The intuition for this result comes from the information that investors infer when both analysts issue similar forecasts compared to the case when analysts disagree about the expected firm's earnings. If the established analyst is believed to be of high competency and the information of competent analysts is highly correlated, then the new analyst is better off when his forecast is not accurate but similar to that of the established analyst than being more accurate while disagreeing with the established analyst's forecast.

REMARK **2.** *Reputation (non-)monotonicity is independent of the distribution of the new analyst's competency, θ_n.*

Proposition 1 suggests that the new analyst's reputation is not necessarily monotonic in his forecast accuracy. Moreover, according to the remark above, this result is independent of investors' initial beliefs about the new analyst's competency. To illustrate this point let $\gamma_n = \frac{1-\theta_n}{\theta_n}$ denote the ratio of incompetent to competent new analysts. Any change in γ_n does not affect the relative ranking of ex-post reputation, e.g., $\tilde{\theta}_n(f_n^m, f_n^v, \pi^m)$ compared to $\tilde{\theta}_n(f_n^v, f_n^v, \pi^m)$. Changes in the ex-post reputations relative ranking can only arise from the additional information that arises from the established analyst's forecast, which depends on his perceived competency and the correlation of analysts' signals but is independent of the new analyst's initial reputation, θ_n.

The assumption of correlated signals for competent analysts is similar to that of Scharfstien and Stein (1990), where competent managers, who care about their reputation, observe fully correlated information and invest sequentially. They show that managers herd in their investment decisions. That is, they ignore their private information to appear as being well informed. In contrast, in this paper of simultaneous forecasting game, analysts issue forecasts according to their information. Note, however, that the lack of monotonicity arises because of the correlation between the signals of competent analysts, which is in contrast to the prevalent assumption in prior literature of (conditionally) independent signals, see for example Trueman (1994).[14] If analysts' information is independent of each other, then an analyst's reputation is independent of the other analysts' forecasts and, therefore, monotonic in the forecast accuracy.

REMARK **3.** *If* $\rho = 0$ *then* $\tilde{\theta}_i(f_i, f_{-i}, \pi) = \tilde{\theta}_i(f_i, \pi)$

The correlation of information of competent analysts may be due to common industry knowledge or because of similarities in the models that analysts use to predict firms' earnings. Proposition 1 suggests that investors' beliefs that determine analysts' reputation and their compensation may develop differently for analysts who cover different

[14]Banerjee (2020) analyzes a model with correlated signals, however, in his model analysts do not differ in their abilities, and accordingly, they do not maximize their future reputation.

firms. Thus, analysts of varying abilities may expect to develop different reputations depending on which of the firms they are assigned to cover.

CLAIM **1.** *A competent analyst is expecting to gain a better reputation when covering a firm that is also covered by other analysts than covering a firm exclusively, and vice versa for an incompetent analyst. Formally, Let A be a firm covered by two analysts and B a firm covered by a single analyst, then*

$$\mathbb{E}[\tilde{\theta}_{n,B}(f_{n,B}, \pi_B)|n\ is\ good] < \mathbb{E}[\tilde{\theta}_{n,A}(f_{n,A}, f_{e,A}, \pi_A)|n\ is\ good]$$
$$\mathbb{E}[\tilde{\theta}_{n,A}(f_{n,A}, f_{e,A}, \pi_A)|n\ is\ bad] < \mathbb{E}[\tilde{\theta}_{n,B}(f_{n,B}, \pi_B)|n\ is\ bad]$$

Proof: See Appendix.

According to Claim 1, a competent analyst is better off when he is assigned to cover the same firm as the established one, as opposed to being the only analyst covering a firm, and vice versa for an incompetent one. Analysts may have some influence on which firm they cover. The next Section considers the possibility that some analysts can choose the firm they cover.

4 Endogenous coverage decisions

In this section, I consider analysts' decisions as to which firm to cover. Analysts may have no discretion as to which firm they may cover, i.e., they are exogenously assigned to cover a particular firm. Exogenous coverage may be due to, for example, cases where brokerage houses seek brokerage business or because their clients are interested in specific firms. On the other hand, some analysts can influence the decision of which firms they cover. Lee and So (2017) provide empirical evidence that analysts initiate coverage based on private information, while McNichols and O'Brien (1997) suggest that terminating coverage may be due to private information. Accordingly, I assume that with probability q an analyst has full discretion over which firm he covers, while with probability $1 - q$ he is assigned to either firm A or B with equal probability.[15] I further assume that firm A is covered by an established analyst, e, while firm B has no prior analyst coverage.

The primary consideration of this section is whether analysts of different abilities differ in their coverage decisions. In such a case, investors must take the analyst's coverage decision into account when they assess his ability.

The updating process is then two-fold; first, investors update their prior beliefs about the analyst's ability based on the firm he covers, even before he issues his forecast. Based on analysts' forecasts and earnings realizations, investors update their beliefs about an analyst's competency once again.

The analysis focuses on a new analyst, n, who has discretion over the firm he covers. A strategy for the analyst is comprised of the firm he chooses to cover, $j \in \{A, B\}$, and the forecast he issues, $f_{n,j} \in \mathcal{F}$. Note that an analyst first chooses which firm to cover, and observes a signal about that firm's earnings only after he decides to follow that firm.

Because of the sequential nature of the game, I adjust the equilibrium concept to Perfect Sequential Equilibrium (henceforth *equilibrium*), as in Grossman and Perry (1986). Now, an equilibrium is comprised of the new analyst's strategy, $\sigma_i(s_i) = (j, f_{i,j})$, and investors' beliefs regarding his ability, $\tilde{\theta}_i(Info)$, where investors' information set is comprised of the firm that the new analyst covers, his earnings forecast, the established analyst's forecast, when it exists, and the firm's earnings. Note that if $q < 1$, then the new analyst's initiation of coverage does not entirely reveal whether he was exogenously assigned to cover that firm or elected to cover that firm. Hence, only in the case of $q = 1$

[15] The assumption of equal probability is for convenience only; assuming any fixed probability to be less than one will not affect the results. Also, recall that an analyst can cover at most one firm.

I specify the off-equilibrium beliefs regarding the firm that the new analyst covers. In the case of $q < 1$, covering either of the firms is on the equilibrium path, and thus does not require off-equilibrium beliefs.

To simplify the analysis, I first note that under the assumption that $\alpha = \frac{1}{2}$, in equilibrium, analysts issue their forecasts according to their signals, regardless of the firm they choose to cover.

REMARK **4.** *Let Assumption 1 hold. In equilibrium, for any firm j that the new analyst covers*

$$f_{n,j}(s_{n,j}) = \begin{cases} f^v_{n,j} & if \quad s_{n,j} = s^v_{n,j} \\ f^m_{n,j} & if \quad s_{n,j} = s^m_{n,j} \end{cases}$$

Proof: See Appendix.

The reason that analysts issue forecasts according to their signals, as for the exogenous coverage case, is the two-stage updating process. Initially, investors update their beliefs about the analyst type based on the firm he covers, i.e., they hold new (posterior) beliefs about his ability, and then use these posterior beliefs as the basis for another update based on the earnings forecasts and realization. In other words, the analyst's choice of the forecast he issues comes after investors have embedded the analyst's choice of firm coverage into their beliefs, and so, the forecast is independent of the coverage decision.

I commence the analysis with the case of $q = 1$, i.e., the new analyst has full discretion over the firm he covers. The next proposition suggests that in this case, in equilibrium, both types of analysts pool together to cover the same firm as the established analyst.

Proposition 2 (A pooling equilibrium)**.**

Let $q = 1$, then the new analyst always chooses to cover firm A, regardless of his type, and the off-equilibrium beliefs are given by $\tilde{\theta}_{n,B}(information) = 0$.

Proof: See Appendix.

When the new analyst chooses the firm he covers for sure, then only a pooling equilibrium exists where both types choose to cover firm A, the firm already covered by the established analyst. The intuition behind this result is simple: When the new analyst has full discretion over coverage, if analysts of different types would differ in their coverage choices, investors can identify the analyst's type for sure. Hence, a less competent analyst must pool together with a competent analyst so that a pooling equilibrium can exist. The only pooling equilibrium that holds under the Grossman-Perry refinement is the one where both types cover firm A, since in such a case, only an incompetent analyst has the incentive to deviate, but does not do so because of the off-equilibrium beliefs. The case where both types cover firm B cannot hold in equilibrium since there are no off-equilibrium beliefs that can support such an equilibrium. Such off-equilibrium beliefs must induce the high type analyst not to deviate, which is impossible under the Grossman-Perry refinement.

I now turn to analyze the case of $q < 1$, i.e., where, with some probability (less than one), the new analyst may choose the firm he covers but may also be exogenously assigned to cover each of the firms (with the complementary probability). Therefore, when investors observe the new analyst coverage, they cannot discern his type for sure, even in a separating equilibrium. However, investors update their beliefs about the new analyst's ability based on the firm he covers. The next proposition characterizes the equilibrium in this case.

Proposition 3 (Separating equilibria)**.**

Let $q < 1$. There exists q^* such that:

- *if $q < q^*$, then, in a pure strategy separating equilibrium, a competent analyst covers firm A while an incompetent analyst covers firm B.*

- *if $q^* < q < 1$, then, in equilibrium, a competent analyst covers firm A (with probability 1) while an incompetent analyst mixes between the two firms.*

Proof: See Appendix.

When some analysts can choose the firm they cover while for others coverage is exogenously determined, analysts of different abilities differ in their coverage choices, at least with positive probability. Analysts differ in their coverage decisions because of the ability of incompetent analysts to choose the firm they cover without fully revealing their lack of competency. If the probability of discretion for coverage is small, i.e., only a small fraction of the analysts can choose which firm to cover, then the effect of analyst coverage on investors' beliefs regarding his ability (before he issues a forecast) is limited. Hence, an incompetent analyst chooses to cover firm B while a competent analyst covers firm A. Differences in coverage decisions of analysts of varying abilities affect investors' beliefs before analysts issue any forecast.

Corollary 1. *Before forecasts are issued, an analyst has a better reputation if he covers firm A than if he covers firm B.*
That is, $\tilde{\theta}_n(j = B) < \tilde{\theta}_n(j = A)$.

According to Proposition 4, the probability that the new analyst is competent is larger if he covers firm A than when he covers firm B before he issues an earnings forecast. The intuition lies in the possibility that the analyst had discretion over which firm to cover; A competent analyst will choose to cover firm A while an incompetent one will cover firm B. Thus, an analyst is more likely to be competent if he starts covering the same firm as others than one who initiates coverage for a firm without prior analysts' coverage.

When it is highly likely that analysts have the discretion to choose the firm they cover, a pure strategy equilibrium does not exist. On the one hand, investors assign a high weight to the possibility that the new analyst had discretion over coverage; while on the other hand, there is some (perhaps small) probability that the analyst was assigned to the firm he covers. If the new analyst can choose the firm he covers, then a competent one always prefers to cover firm A. An incompetent analyst has to balance his incentive to cover firm B, where he expects a better reputation based on his forecast, against the incentive to pretend to be competent by covering firm A. In equilibrium, an incompetent analyst mixes between coverage of firm A or B, and thus expects to achieve the same reputation from covering each of the firms. Nonetheless, before the new analyst issues his forecast, the probability that he is of high ability is higher when he covers firm A than when he covers firm B, as was in the pure strategy case.

Remark **5.** *The equilibrium coverage choices of analysts of different abilities are independent of the new analyst's competency distribution,* θ_n.

According to Proposition 3, analysts of different abilities differ in their coverage choices; these choices are independent of the new analyst's initial reputation, θ_n. The intuition for this general result is that analysts are conscious of their abilities. According to Claim 1, a competent analyst is better off when covering firm A because of the excess information arising when covering that firm, and vice versa for an incompetent analyst, regardless of the new analyst's competency distribution. Therefore, the new analyst's coverage decision is independent of the prior distribution of types but rather depends on the expected information that investors obtain when he covers each of the firms.

5 Forecasting bias

Prior literature argues that analysts' earnings forecasts might not reveal all the information analysts possess because analysts may bias their forecasts. In this section, I relax Assumption 1 to consider the case of forecasting bias and the effect of analysts' coverage decisions on such a bias.

Let Δ be a probability measure over $\mathcal{F}$, and let $\sigma_i^k(s_i^k) = Pr(f_i^k|s_i^k)$.[16] An analyst issues a *truthful* forecast if $\sigma_i^k(s_i^k) = 1$, i.e., if he issues a forecast according to his signal. An analyst's forecast is *biased* if $\sigma_i^k(s_i^k) < 1$, i.e., the analyst issues a forecast that does not corresponds to his signal with positive probability. A truthful forecast is a one-to-one mapping from signals to forecasts; thus, it enables investors to deduce the information an analyst observed from the forecast he issues. Although investors learn the underlying signal for sure, the accuracy of the signal is unknown because it depends on the (unobserved) analyst type. In contrast, a biased forecast is a stochastic mapping from signal to forecast, and thus negatively affects the information that investors may learn. The next proposition characterizes the parameter set under which analysts issue truthful or biased forecasts.

Proposition 4 (Truthful and biased forecasts)**.**

1. *Let $\alpha \leq b$. The equilibrium remains as in Lemma 0.*

   That is, analysts issue truthful forecasts regardless of their type.

2. *Let $b < \alpha$.*

   (i) *There does not exist a pure strategy equilibrium of truthful forecasts, i.e., an equilibrium such that $\sigma_i^k(s_i^k) = 1$.*

   (ii) *There exists an equilibrium such that a competent analyst issues a truthful forecast, while an incompetent analyst mixes after observing the signal s_i^v and*

[16] I forgo the firm's index for the moment.

forecasts truthfully after observing the signal s_i^m. *That is,* $\sigma_i^v(s_i^v|n \text{ is bad}) < 1$, *and* $\sigma_i^k(s_i^k) = 1$ *if* n *is good or* $k \neq v$.

Proof: See Appendix.

Proposition 4 suggests that a competent analyst always issues a forecast according to his signal while an incompetent analyst biases his forecast when moderate earnings, π^m, are more prevalent, i.e., when $\frac{1}{2} < \alpha$, and his information is of low accuracy, i.e., low values of b. Whether or not an incompetent analyst issues a biased forecast is independent of the firm he covers. The reason for forecasting bias arises from the weak information an incompetent analyst possesses; when $b < \alpha$, an incompetent analyst attributes higher probability to moderate earnings, π^m, than to extreme earnings, π^v, even after observing an extreme signal, s_i^v, regardless of the firm he covers. Since analysts aim to appear well informed by issuing a forecast that corresponds to the firm's earnings, an incompetent analyst has an incentive to issue a forecast of f_i^m even after observing the signal s_i^v. On the other hand, when such a bias exists, an analyst is more likely to be competent when he issues an extreme forecast, f_i^v. As a result, an incompetent analyst may benefit from issuing an extreme forecast, and so, mixes between the two possible forecasts.

The reasons that induce incompetent analysts to bias their forecasts are present for any firm the analysts cover. According to Proposition 4, when an incompetent analyst's information is relatively inaccurate, i.e., $b < \alpha$, he may bias his forecast regardless of the firm he covers. Nevertheless, an incompetent analyst expects to build a different reputation when he covers firm A or B. Therefore, the firm that an incompetent analyst covers affects the magnitude of the bias, as suggested by the next proposition.

Proposition 5 (Coverage and the magnitude of bias)**.**

An incompetent analyst biases his forecast more when he covers firm A *than when he covers firm* B. *That is,* $\sigma_{i,B}^v(s_i^v|n \text{ is bad}) < \sigma_{i,A}^v(s_i^v|n \text{ is bad})$.

Proof: See Appendix.

According to Proposition 9, I expect an incompetent analyst to bias his forecast more when he covers the same firm as the established analyst than in the case he is the only one who follows the firm. The intuition behind Proposition 9 is as follows: Incompetent analysts bias their forecasts in case the signal they observe is not sufficient to change their perception for the more likely state, i.e., the firm's earnings, independently of the firm they cover. However, the reputation they gain from providing a forecast that reflects the firm's earnings depends on the firm they cover. When the new analyst covers firm A, together with the established ones, he gains more if he also issues the same forecast as the established analyst issues. An incompetent analyst expects the established analyst to issue a forecast according to the more likely state, i.e., π^m. Thus, when covering firm A, an incompetent analyst has a greater incentive to bias his forecast, that is, to issue a forecast according to the more likely event of moderate earnings.

6 Discussion and empirical implications

The previous sections' results may explain the mixed empirical findings in past literature and provide new insight for future empirical tests. In the following, I explore a few empirical implications. The first implication focuses on the effect of analyst forecast accuracy on career outcomes. While several empirical papers analyzed the relationship between analyst forecast accuracy and career outcomes, which they use as a proxy for reputation, mixed results were found. For example, Hong and Kubik (2003) show that analysts with more accurate forecasts than others are more likely to move on to more prestigious brokerage houses; on the other hand, Makhil et al. (2003) did not find such an effect. The results provide a possible explanation to the mixed results; reputation may be non-monotonic in forecast accuracy, depending on the (conditional) correlation of analysts' information. Proposition 1 suggests that reputation is monotonic in forecast accuracy if the correlation between competent analysts' information is not too high. Thus, to examine the effect of forecast accuracy on future career outcomes, one must first control for the correlation between analysts' information, for example, through the correlation of their previous forecasts. It is important to note that most empirical studies do not use the absolute forecast accuracy but rather the relative forecast accuracy. Nonetheless, the correlation of information still plays a significant role in analysts' reputations. To illustrate this point, let us assume a firm that is covered by three analysts. Consider the case where two of the analysts provide similar forecasts, which are less accurate (ex-post) than the forecast of the third. If competent analysts' information is highly correlated, then the two analysts who provide similar forecasts should experience a better outcome than the third one who is more accurate.

The second implication arises from the notion that analysts of different abilities are likely to cover different firms if they have (some) discretion over coverage. The results suggest that the career outcomes of analysts who initiate coverage for firms with more existing coverage should outperform those who cover firms on an exclusive basis. While I am not aware of an empirical study that answers this question directly, Crawford et al. (2012) may provide a shred of initial evidence for this paper's new insight; they show that the information content of an analyst who is the first to initiate coverage for a firm differs from that of an analyst who covers a firm with established coverage. An analyst who covers a firm exclusively provides general information about the firm, thus increases the synchronicity with the industry earnings. In contrast, an analyst who covers a firm where prior coverage exists provides firm-specific information that reduces this synchronicity. It is an empirical question of whether analysts who are the first ones to initiate coverage of a firm, experience different career outcomes than those who initiate coverage of firms with prior analyst coverage.

Lastly, I have shown that forecasting bias varies with analyst coverage; less competent analysts are likely to bias their forecasts toward their prior expectations when covering firms with additional analyst coverage more than they would when covering a firm exclusively. Moreover, bold forecasts are more likely to be issued by more competent analysts covering the same firm as others. Thus, I expect bold forecasts to be more accurate for firms with greater coverage. Accordingly, I can expect a more significant price impact of bold forecasts for firms with more coverage. Whether bold forecasts have different price impact for firms with more analyst coverage as compared to firms with single analyst coverage seems to be an interesting empirical question.

7 Concluding Remarks

It has been demonstrated in past literature that sell-side analysts differ in abilities. Forecast accuracy is commonly used, in the theoretical and empirical literature, to identify differences in analysts' abilities, which is not surprising because of its observability. However, analyst initiations of coverage, which is another analyst's observable action, has been overlooked by theoretical and empirical studies when trying to differentiate competent analysts from less competent ones. In this paper, I study how coverage decisions vary for analysts of heterogeneous abilities and demonstrate that initiation of coverage may provide information about analysts' abilities. An essential feature in the model is the conditional correlation of information of more able analysts. Many theoretical and empirical studies focus on the monotonic relations between forecast accuracy and reputation. In contrast, I show that correlated information may result in non-monotonic relations of forecast accuracy and reputation. This result might suggest an explanation for the mixed results in the empirical literature that examines the effect of forecast accuracy on analysts' careers. Lastly, I offer additional insight into the tendency of less competent analysts to bias their forecasts, illustrating the impact of analysts' coverage on forecasting bias.

Appendix A - Proofs

Proof of Remark 1: Assume that a good type analyst issues a forecast from the set $\mathcal{F}^{good}$ while a bad type analyst issues a forecast from the set $\mathcal{F}^{bad}$ and that there exists a forecast $\tilde{f}$ such that $\tilde{f} \in \mathcal{F}^{bad}$ but $\tilde{f} \notin \mathcal{F}^{good}$: Then issuing forecast $\tilde{f}$ must result in $Pr(\text{type is } good|\tilde{f}) = 0$. Thus, in equilibrium, a bad type analyst will never issue forecast $\tilde{f}$. □

It is useful to present the new analyst's reputation for a given forecast, firm's earnings, and, if it exists, the established analyst's forecast, i.e., $\tilde{\theta}_n(f_n(\sigma_n), \pi)$ and $\tilde{\theta}_n(f_n(\sigma_n), f_e(\sigma_e), \pi)$, when analysts use pure strategies. Without loss of generality, we assume that the pure strategies are such that f_i^m (f_i^v) corresponds to s_i^m (s_i^v); We adjust the reputation to include mixed strategies and specify these mixed strategies to prove the results of Section 5.

Assume analysts use pure strategies of the form

$$f_i(s_i) = \begin{cases} f_i^v & if \quad s_i = s_i^v \\ f_i^m & if \quad s_i = s_i^m \end{cases}$$

The ex-post reputation for a new analyst that covers a firm exclusively is given by:

$$\begin{aligned} \tilde{\theta}_n(f^m, \pi = \pi^m) = \tilde{\theta}_n(f^v, \pi = \pi^v) &= \frac{g\theta}{g\theta + b(1-\theta)} \\ \tilde{\theta}_n(f^m, \pi = \pi^v) = \tilde{\theta}_n(f^v, \pi = \pi^m) &= \frac{(1-g)\theta}{(1-g)\theta + (1-b)(1-\theta)} \end{aligned}$$

The ex-post reputation for a new analyst that covers a firm that is also covered by the established analyst is given by:

$$\begin{aligned}
\tilde{\theta}_n(f_n^m, f_e^m, \pi^m) &= \tilde{\theta}_n(f_n^v, f_e^v, \pi^v) = \\
&= \tfrac{\theta_n(\theta_e(\rho g+(1-\rho)g^2)+(1-\theta_e)gb)}{\theta_n(\theta_e(\rho g+(1-\rho)g^2)+(1-\theta_e)gb)+(1-\theta_n)(\theta_e bg+(1-\theta_e)b^2)} \\
\tilde{\theta}_n(f_n^m, f_e^v, \pi^m) &= \tilde{\theta}_n(f_n^v, f_e^m, \pi^v) = \\
&= \tfrac{\theta_n(\theta_e(1-\rho)g(1-g)+(1-\theta_e)g(1-b))}{\theta_n(\theta_e(1-\rho)g(1-g)+(1-\theta_e)g(1-b))+(1-\theta_n)(\theta_e b(1-g)+(1-\theta_e)b(1-b))} \\
\tilde{\theta}_n(f_n^m, f_e^m, \pi^v) &= \tilde{\theta}_n(f_n^v, f_e^v, \pi^m) = \\
&= \tfrac{\theta_n(\theta_e(\rho(1-g)+(1-\rho)(1-g)^2)+(1-\theta_e)(1-g)(1-b))}{\theta_n(\theta_e(\rho(1-g)+(1-\rho)(1-g)^2)+(1-\theta_e)(1-g)(1-b))+(1-\theta_n)(\theta_e(1-b)(1-g)+(1-\theta_e)(1-b)^2)} \\
\tilde{\theta}_n(f_n^m, f_e^v, \pi^v) &= \tilde{\theta}_n(f_n^v, f_e^m, \pi^m) = \\
&= \tfrac{\theta_n(\theta_e(1-\rho)g(1-g)+(1-\theta_e)(1-g)b)}{\theta_n(\theta_e(1-\rho)g(1-g)+(1-\theta_e)(1-g)b)+(1-\theta_n)(\theta_e(1-b)g+(1-\theta_e)(1-b)b)}
\end{aligned}$$

Proof of Lemma 1: Analysts maximize their reputation. That is

$$Max_{f_n}\mathbb{E}[\tilde{\theta}_n(f_n, \pi)|s_n]$$

if the new analyst covers a firm exclusively, and

$$Max_{f_n}\mathbb{E}[\tilde{\theta}_n(f_n, f_e, \pi)|s_n]$$

if he covers the same firm as the established analyst. Where the conditional expectation is over earnings and, if it exists, the established analyst's forecast.

Let us assume that investors believe that analysts issue forecasts according to their signals, so reputation is determined accordingly. We have to show that analysts maximize their reputation when issuing forecasts that reflects their signals.

When an analyst of type $t \in \{good, bad\}$ observes the signal s_n his posterior distribution for the possible firm's earnings is given in (1). For $\alpha = \frac{1}{2}$ we get

$$Pr(\pi^m|s_i^m) = Pr(\pi^v|s_i^v) = p$$
$$Pr(\pi^m|s_i^v) = Pr(\pi^v|s_i^m) = 1 - p$$

where $p \in \{g, b\}$. Recall that $\frac{1}{2} < b$.

First, when the new analyst covers a firm exclusively the expected reputation given a signal s_n is

$$\mathbb{E}[\tilde{\theta}_n(f_n, \pi)|s_n] = Pr(\pi^m|s_n)\tilde{\theta}_n(f_n, \pi^m) + Pr(\pi^v|s_n)\tilde{\theta}_n(f_n, \pi^v)$$

Now, note that for $0 < \theta < 1$, we get $\tilde{\theta}_n(f^{j'}, \pi^j) < \tilde{\theta}_n(f^j, \pi^j)$ where $j \in \{1, v\}$ and $j' \neq j$, since $b < g$.

Second, when the new analyst covers the same firm as the established one, his expected reputation given a signal s_n is

$$\begin{aligned}
&\mathbb{E}[\tilde{\theta}_n(f_n, f_e, \pi)|s_n] = \\
&\quad = Pr(\pi^m|s_n)\mathbb{E}_{f_e}[\tilde{\theta}_n(f_n, f_e, \pi^m)|s_n] + Pr(\pi^v|s_n)\mathbb{E}_{f_e}[\tilde{\theta}_n(f_n, f_e, \pi^v)|s_n] \\
&\quad = Pr(\pi^m|s_n)\big(Pr(s_e^m|s_n, \pi^m)\tilde{\theta}_n(f_n, f_e^m, \pi^m) + Pr(s_e^v|s_n, \pi^m)\tilde{\theta}_n(f_n, f_e^v, \pi^m)\big) \\
&\qquad + Pr(\pi^v|s_n)\big(Pr(s_e^m|s_n, \pi^v)\tilde{\theta}_n(f_n, f_e^m, \pi^v) + Pr(s_e^v|s_n, \pi^v)\tilde{\theta}_n(f_n, f_e^v, \pi^v)\big)
\end{aligned}$$

To show that honest forecasts constitute an equilibrium, we have to show that upon observing the signal s_n^j the new analyst expects higher reputation when issuing a forecast of f_n^j than issuing the forecast $f_n^{j'}$.

Note that

$$\tilde{\theta}_n(f_n^j, f_e^{j'}, \pi^j) < \tilde{\theta}_n(f_n^j, f_e^j, \pi^j) \text{ and } \tilde{\theta}_n(f_n^{j'}, f_e^{j'}, \pi^j) < \tilde{\theta}_n(f_n^j, f_e^j, \pi^j) \tag{4}$$

that is, the new analyst's reputation is higher when both analysts issued accurate forecast compared to 1. the case when only the established analyst was accurate, and 2. when both issued inaccurate forecasts.

In addition note that

$$\tilde{\theta}_n(f_n^{j'}, f_e^j, \pi^j) < \tilde{\theta}_n(f_n^{j'}, f_e^{j'}, \pi^j) \text{ and } \tilde{\theta}_n(f_n^{j'}, f_e^j, \pi^j) < \tilde{\theta}_n(f_n^j, f_e^{j'}, \pi^j) \tag{5}$$

That is, the new analyst's reputation is lower when issuing an inaccurate forecast while the established analysts issues an accurate one compared to: (i) the case were both forecasts are inaccurate, and (ii) when the new analyst's forecast is accurate while the established one is inaccurate.

Combining the inequalities from (4) and (5) we get

$$|\tilde{\theta}_n(f_n^{j'}, f_e^{j'}, \pi^j) - \tilde{\theta}_n(f_n^j, f_e^{j'}, \pi^j)| < \tilde{\theta}_n(f_n^j, f_e^j, \pi^j) - \tilde{\theta}_n(f_n^{j'}, f_e^j, \pi^j) \tag{6}$$

Also, if a bad type analyst issues a forecast according to his signal, a good type analyst will also do so. Now, for any signal s_n, for a bad type analyst we get

$$Pr(s_e^{j'}|\pi^j, s_n) < Pr(s_e^j|\pi^j, s_n)$$

where $j' \neq j$.

Thus, together with the fact that $Pr(\pi^{j'}|s_n^j) < Pr(\pi^j|s_n^j)$ and (6) we get that upon observing the signal s_n^j the new analyst expects higher reputation when issue a forecast of f_n^j than when issuing the forecast $f_n^{j'}$.

Note that a pure strategy equilibrium is more informative than any mixed strategy equilibrium. $\square$

Proof of Proposition 1: According to Lemma 1 analysts issue forecasts according to their signals.

(i) When the new analyst covers a firm exclusively, his reputation is monotonic in his forecast accuracy. Formally,

$$\begin{aligned}
\tilde{\theta}_n(f_n^{j'}, \pi^j) &< \tilde{\theta}_n(f_n^j, \pi^j) \\
\frac{(1-g)\theta_n}{(1-g)\theta_n + (1-b)(1-\theta_n)} &< \frac{g\theta_n}{g\theta_n + b(1-\theta_n)} \\
\iff b &< g
\end{aligned}$$

(ii) When the new analyst covers the same firm as the established one his reputation is not monotonic when the reputation from being inaccurate is higher than the reputation arising from an accurate forecast. This is only possible in the case

$$\tilde{\theta}_n(f_n^j, f_e^{j'}, \pi^j) < \tilde{\theta}_n(f_n^{j'}, f_e^{j'}, \pi^j)$$

$$\frac{\theta_n(\theta_e(1-\rho)g(1-g)+(1-\theta_e)g(1-b))}{\theta_n(\theta_e(1-\rho)g(1-g)+(1-\theta_e)g(1-b))+(1-\theta_n)(\theta_e b(1-g)+(1-\theta_e)b(1-b))}$$

$$< \quad \frac{\theta_n(\theta_e(\rho(1-g)+(1-\rho)(1-g)^2)+(1-\theta_e)(1-g)(1-b))}{\theta_n(\theta_e(\rho(1-g)+(1-\rho)(1-g)^2)+(1-\theta_e)(1-g)(1-b))+(1-\theta_n)(\theta_e(1-b)(1-g)+(1-\theta_e)(1-b)^2)}$$

Simplifying this inequality we get that it holds, for any b, g, and θ_n, if and only if $\frac{(g-b)(1-b)}{g(1-b)-b(g-b)} < \theta_e$ and $\frac{(g-b)(1-b-\theta_e(g-b))}{\theta_e g(1-g)} < \rho$.

$\square$

Proof of Remark 2: Note that if $\rho = 0$ then

$$Pr(s_n|s_e, \pi) = \frac{Pr(s_n \cap_e \cap \pi)}{Pr(s_e \cap \pi)} = \frac{Pr(s_n|\pi)Pr(s_e|\pi)}{Pr(s_e|\pi)} = Pr(s_n|\pi)$$

Thus, the reputation of the new analyst is independent of the established analyst's forecast. $\square$

Proof of Claim 1: Following Lemma 1, analysts issue forecasts according to their signals. Now,

$$\begin{aligned}
&\mathbb{E}[\tilde{\theta}_{n,B}(f_{n,B}, \pi_B)|\ n\text{'s type}] = \\
&Pr(\pi^m|s_n^j,\ n\text{'s type})\tilde{\theta}_n(f_n^j, \pi^m) + Pr(\pi^v|s_n^j,\ n\text{'s type})\tilde{\theta}_n(f_n^j, \pi^v)
\end{aligned}$$

and

$$\begin{aligned}
&\mathbb{E}[\tilde{\theta}_{n,A}(f_{n,A}, f_{e,A}, \pi_A)|\ n\text{'s type}] = \\
&Pr(\pi^m|s_n^j,\ n\text{'s type})\mathbb{E}_{s_e|\pi^m}[\tilde{\theta}_n(f_n, f_e(s_e), \pi^m)] \\
&\qquad\qquad + Pr(\pi^v|s_n^j,\ n\text{'s type})\mathbb{E}_{s_e|\pi^v}[\tilde{\theta}_n(f_n, f_e(s_e), \pi^v)]
\end{aligned}$$

where

$$\begin{aligned}
&\mathbb{E}_{s_e|\pi^j, s_n}[\tilde{\theta}_n(f_n, f_e(s_e), \pi^j)] = \\
&Pr(s_e^j|\pi^j, s_n)\tilde{\theta}_n(f_n, f_e^j, \pi^j) + Pr(s_e^{j'}|\pi^j, s_n)\tilde{\theta}_n(f_n, f_e^{j'}, \pi^j)
\end{aligned}$$

Let us assume, w.l.o.g., that the new analyst observes a signal of s_n^m. From equation (1) we get $Pr(\pi^v|s^m) < Pr(\pi^m|s^m)$, regardless of his type.

Thus, differences in an analyst's expected reputation can only arise from the probabilities $Pr(s_e^j|\pi^j, s_n)$ and $Pr(s_e^{j'}|\pi^j, s_n)$.

From an incompetent analyst's view point

$$Pr(s_e^m|\pi^m, s_n^m) = Pr(s_e^m|\pi^m) = eg + (1-e)b$$

and

$$Pr(s_e^v|\pi^m, s_n^m) = Pr(s_e^v|\pi^m) = e(1-g) + (1-e)(1-b)$$

That is, the probability that the established analyst observes a signal s_e^j is independent of the signal an incompetent analyst observes.
However, for a competent analyst

$$Pr(s_e^m|\pi^m, s_n^m) = e(\rho + (1-\rho)g) + (1-e)b$$

and

$$Pr(s_e^v|\pi^m, s_n^m) = e(1-\rho)(1-g) + (1-e)(1-b)$$

Since,

$$(eg+(1-e)b)\tilde{\theta}_n(f_n^m, f_e^m, \pi^m) + (e(1-g)+(1-e)(1-b))\tilde{\theta}_n(f_n^m, f_e^v, \pi^m) < \tilde{\theta}_n(f_n^m, \pi^m)$$

and

$$(eg+(1-e)b)\tilde{\theta}_n(f_n^m, f_e^v, \pi^v) + (e(1-g)+(1-e)(1-b))\tilde{\theta}_n(f_n^m, f_e^m, \pi^v) < \tilde{\theta}_n(f_n^v, \pi^m)$$

an incompetent analyst is better off covering firm B.

In contrast,

$$\tilde{\theta}_n(f_n^m, \pi^m) < (e(\rho + (1-\rho)g) + (1-e)b)\tilde{\theta}_n(f_n^m, f_e^m, \pi^m)$$
$$+(e(1-\rho)(1-g) + (1-e)(1-b))\tilde{\theta}_n(f_n^m, f_e^v, \pi^m)$$

and

$$\tilde{\theta}_n(f_n^v, \pi^m) < (e(1-\rho)g + (1-e)b)\tilde{\theta}_n(f_n^m, f_e^v, \pi^v)$$
$$+(e(\rho + (1-\rho)(1-g) + (1-e)(1-b)\tilde{\theta}_n(f_n^m, f_e^m, \pi^v)$$

Thus, a competent analyst is better off when covering firm A. □

Proof of Remark 3: Follows immediately from the sequential nature of the Bayesian updating about the analyst's type. □

Proof of Proposition 2: Let $q = 1$. First, note that the equilibrium must be a pooling equilibrium; if a separating equilibrium exists, investors identify the underlying type of the analyst with probability 1, and thus, an incompetent analyst would deviate to cover the other firm.

There are two possible pooling equilibria: where both types cover firm A, or B. When analysts of both types cover firm A and the off-equilibrium beliefs are such that $Pr(\text{type is } good | j \text{covers } B) = 0$, then each of the possible types is worse off if the analyst deviates to cover firm B. The off-equilibrium beliefs satisfy the Grossman-Perry refinement criterion since even if both types were assumed to deviate, a competent analyst would not deviate because, according to Claim 1, he is better off covering firm A.

The case where both types cover firm B cannot hold in equilibrium. Assume that both types cover firm B; then according to the Grossman-Perry criterion, any off-equilibrium beliefs must include a deviation by competent analysts because if both types where to deviate, a competent analyst is better-off doing so. However, for such off-equilibrium beliefs a competent analyst would deviate and cover firm A, in contrast to the equilibrium strategies. $\square$

Proof of Proposition 3: I start by showing that a pure strategy separating equilibrium can hold only for some q^*.

First note that the only possible separating equilibrium is when a competent analyst covers firm A while an incompetent one covers firm B. The opposite case cannot hold in equilibrium for the reason I show in the followings.

Since $q < 1$ coverage of either of the firms is on the equilibrium path, and thus, no off-equilibrium beliefs should be specified.

Let $\theta_{n,j} = Pr(n \text{ is } good | n \text{ covers firm } j)$. Under this pure strategy separating equilibrium we get

$$\theta_{n,j} = \begin{cases} \frac{1}{2}(1+q) & if \quad j = A \\ \frac{1}{2}(1-q) & if \quad j = B \end{cases}$$

Let $U_{t,j} = \mathbb{E}_{s,\pi}[\tilde{\theta}_n(\cdot)|\theta_{n,j}, t]$ denote the expected reputation for the new analyst of type t who covers firm j.

First, note that $U_{t,A}$ is increasing in q while $U_{t,B}$ is decreasing in q.

Second, note that in this separating equilibrium, for any q a competent analyst it is always better off when covering firm A than B, because $q = 0$ is equivalent to the exogenous assignment case.

Now, for an incompetent analyst, for $q = 0$ he expects the same reputation as in the exogenous case, so $U_{bad,A} < U_{bad,B}$. For $q \to 1$, under the proposed equilibrium strategies, we get $0 = U_{t,B} < U_{t,A} = 1$. Since $U_{t,j}$ is continuous in q, there exists q^*

such that for any $q \leq q^*$ we have $U_{t,A} < U_{t,B}$, so a bad type analyst has no profitable deviation, and the pure strategies equilibrium holds.

For $q^* < q$ the suggested pure strategy equilibrium cannot hold since an incompetent analyst always has a profitable deviation.

Assume $q^* < q$. I will show that there exists an equilibrium where a competent analyst covers firm A for sure while an incompetent analyst mixes between the two firms.

Let γ denote the probability that an incompetent analyst covers firm A. Note that, in equilibrium, investors beliefs are consistent with γ. That is, $\theta_{n,j}(\gamma)$ is determined according to the equilibrium value of γ. Formally,

$$\theta_{n,j}(\gamma) = \begin{cases} q(\theta + (1-\theta)\gamma) + \frac{1}{2}(1-q) & if \quad j = A \\ q(1-\theta)(1-\gamma) + \frac{1}{2}(1-q) & if \quad j = B \end{cases}$$

Assume $\gamma = 1$. that is, an incompetent analyst covers firm A for sure. Since also a competent analyst cover firm A, analysts expect the same payoff as in the exogenous case. Thus, according to Lemma 1 we have $U_{bad,A} < U_{bad,B}$.

Assume $\gamma = 0$. that is, an incompetent analyst covers firm B for sure. In other words, analysts strategies are the same as under the pure strategy equilibrium above. Since, $q^* < q$ we know that $U_{bad,B} < U_{bad,A}$.

One can observe that $\theta_{n,j}(\gamma)$ is continuous in γ, thus, also $U_{bad,B}$ and $U_{bad,A}$ are continuous in γ. Therefore, there exists a unique γ such that $U_{bad,B} = U_{bad,A}$, which is the equilibrium level of mixing.

Note that the mix strategy equilibrium above is more informative than a pure strategy equilibrium where the new analyst covers firm A for sure. The reason for a mix strategy equilibrium being more informative is the information investors learn about the new analyst type. $\square$

Proof of Corollary 1: Follows directly from the proof of Proposition 3. $\square$

Proof of Proposition 4: Assume $\frac{1}{2} < \alpha$.

After observing a signal s_n the new analyst updates the earnings distribution according to (1). For an incompetent analyst who observes a signal s_n^v we get

$$Pr(\pi^m|s_n^v) < Pr(\pi^v|s_n^v) \Leftrightarrow \alpha < b$$

and

$$Pr(\pi^v|s_n^v) < Pr(\pi^m|s_n^v) \Leftrightarrow b < \alpha$$

Similarly,

$$Pr(s_e^m|s_n^v) < Pr(s_e^v|s_n^v) \Leftrightarrow \alpha < b \qquad \text{and } Pr(s_e^v|s_n^v) < Pr(s_e^m|s_n^v) \Leftrightarrow b < \alpha$$

Assume $\alpha < b$; then there exists an equilibrium as in Lemma 1. That is, each analyst issues a forecast according to the more likely state, which is according to the signal he observes.

Assume $b < \alpha$.

First, note that truthful strategies cannot constitute an equilibrium. Assume that investors beliefs are such that each analyst issues a truthful forecast, then the ex-post reputations $\tilde{\theta}_n(f_n, \pi)$ and $\tilde{\theta}_n(f_n, f_e, \pi)$ are as specified above (Refer to the beginning of the Appendix). Since $b < \alpha$, from the point of view of an incompetent analyst who observes the signal s_n^v, the more likely state is π^m. Therefore, he is better off deviating to issue the forecast f_n^m.

To show the mixed strategy equilibrium described in Proposition 4, one first has to adjust the ex-post reputation equations in the beginning of the Appendix. Assume that analysts issue forecasts according to the strategies specified in Proposition 4. Without loss of generality, assume that the established analyst is known to be competent. Recall that an incompetent analyst who observes the signal s_n^v, issues the forecast f_n^v with probability σ_n^v; to avoid excessive notation let us denote $\sigma_n^v = \sigma$.

The ex-post refutation for an analyst who covers firm B is given by

$$\begin{aligned}
\tilde{\theta}_n(f^m, \pi = \pi^m) &= \frac{g\theta}{g\theta + (1-\theta)(b + (1-b)(1-\sigma))} \\
\tilde{\theta}_n(f^m, \pi = \pi^v) &= \frac{(1-g)\theta}{(1-g)\theta + (1-\theta)(b(1-\sigma) + 1 - b)} \\
\tilde{\theta}_n(f^v, \pi = \pi^v) &= \frac{g\theta}{g\theta + (1-\theta)b\sigma} \\
\tilde{\theta}_n(f^v, \pi = \pi^m) &= \frac{(1-g)\theta}{(1-g)\theta + (1-\theta)(1-b)\sigma}
\end{aligned}$$

Let $p^v = Pr(\pi^v|s_n^v)$ and $p^m = Pr(\pi^m|s_n^v)$. The expected reputation for an incompetent analyst who observes the signal s_n^v and issues the forecast f_n^v is then

$$U_{mix}(f^v) \equiv p^m\tilde{\theta}_n(f^v, \pi = \pi^m) + p^v\tilde{\theta}_n(f^v, \pi = \pi^v)$$

Onthe other hand, the expected reputation for an incompetent analyst who observes the signal s_n^v, issue the forecast f_n^m is

$$U_{mix}(f^m) \equiv p^m\tilde{\theta}_n(f^m, \pi = \pi^m) + p^v\tilde{\theta}_n(f^m, \pi = \pi^v)$$

Now, $U_{mix}(f^v)$ is decreasing in σ and $U_{mix}(f^m)$ is increasing in σ. Furthermore,

$$\lim_{\sigma\to 0} \frac{g\theta}{g\theta + (1-\theta)} = U_{mix}(f^m) < U_{mix}(f^v) = 1$$

and

$$\lim_{\sigma\to 1} U_{mix}(f^v) < U_{mix}(f^m)$$

because $p^v < p^m$ and $\frac{(1-g)\theta}{(1-g)\theta+(1-\theta)(1-b)} < \frac{g\theta}{g\theta+(1-\theta)b\sigma}$

Thus, there exists a unique $\sigma = \sigma_B$ such that $U_{mix}(f^v) = U_{mix}(f^m)$.

The ex-post refutation for an analyst who covers firm A is given by

$$\tilde{\theta}_n(f_n^m, f_e^m, \pi^m) = \frac{\theta_n(\rho g + (1-\rho)g^2)}{\theta_n(\rho g + (1-\rho)g^2) + (1-\theta_n)g(b+(1-b)(1-\sigma))}$$

$$\tilde{\theta}_n(f_n^m, f_e^v, \pi^m) = \frac{\theta_n(1-\rho)g(1-g)}{\theta_n(1-\rho)g(1-g) + (1-\theta_n)(1-g)(b+(1-b)(1-\sigma))}$$

$$\tilde{\theta}_n(f_n^m, f_e^m, \pi^v) = \frac{\theta_n(\rho(1-g)+(1-\rho)(1-g)^2)}{\theta_n(\rho(1-g)+(1-\rho)(1-g)^2)+(1-\theta_n)(1-g)(b(1-\sigma)+(1-b))}$$

$$\tilde{\theta}_n(f_n^m, f_e^v, \pi^v) = \frac{\theta_n(1-\rho)g(1-g)}{\theta_n(1-\rho)g(1-g) + (1-\theta_n)g(b(1-\sigma)+(1-b))}$$

$$\tilde{\theta}_n(f_n^v, f_e^v, \pi^v) = \frac{\theta_n(\rho g + (1-\rho)g^2)}{\theta_n(\rho g + (1-\rho)g^2) + (1-\theta_n)bg\sigma}$$

$$\tilde{\theta}_n(f_n^v, f_e^m, \pi^v) = \frac{\theta_n(1-\rho)g(1-g)}{\theta_n(1-\rho)g(1-g) + (1-\theta_n)b(1-g)\sigma}$$

$$\tilde{\theta}_n(f_n^v, f_e^v, \pi^m) = \frac{\theta_n(\rho(1-g) + (1-\rho)(1-g)^2)}{\theta_n(\rho(1-g) + (1-\rho)(1-g)^2) + (1-\theta_n)(1-b)(1-g)\sigma}$$

$$\tilde{\theta}_n(f_n^v, f_e^m, \pi^m) = \frac{\theta_n(1-\rho)g(1-g)}{\theta_n(1-\rho)g(1-g) + (1-\theta_n)g(1-b)\sigma}$$

Similar to the analysis when the new analyst covers firm B, when the new analyst covers firm A let

$$U^A_{mix}(f^v) \equiv \mathbb{E}_{\pi,f_e}[\tilde{\theta}_n(f^v_n, f_e, \pi)|s^v_n]$$

and

$$U^A_{mix}(f^m) \equiv \mathbb{E}_{\pi,f_e}[\tilde{\theta}_n(f^m_n, f_e, \pi)|s^v_n]$$

Again $U^A_{mix}(f^v)$ is decreasing in σ and $U^A_{mix}(f^m)$ is increasing in σ. Furthermore,

$$\lim_{\sigma\to 0} U^A_{mix}(f^m) < U^A_{mix}(f^v) = 1$$

Let $r^m \equiv Pr(s^m_e|s^v_n)$ and $r^v \equiv Pr(s^v_e|s^v_n)$. Since $p^v < p^m$ also $r^v < r^m$. Therefore,

$$\lim_{\sigma\to 1} U^A_{mix}(f^v) < U^A_{mix}(f^m)$$

Thus, there exists a unique $\sigma = \sigma_A$ such that $U^A_{mix}(f^v) = U^A_{mix}(f^m)$. $\square$

Proof of Proposition 5: Let $U_{mix}(f^m)$ and $U^A_{mix}(f^m)$ denote the expected reputation of an incompetent analyst who issues a forecast f^m_n after observing the signal s^m_n when he covers firm B and A, respectively. Let p^v, p^m, r^v, and r^m be defined as in the proof of Proposition 4. For any σ we have $U_{mix}(f^m) < U^A_{mix}(f^m)$ because $p^v < p^m$ and $r^v < r^m$. In a similar way let $U_{mix}(f^v)$ and $U^A_{mix}(f^v)$. For any σ we have $U^A_{mix}(f^v) < U_{mix}(f^v)$. Because the mixing weights are the solutions to $U_{mix}(f^m) = U_{mix}(f^v)$ and $U^A_{mix}(f^m) = U^A_{mix}(f^v)$, we find that $\sigma_A < \sigma_B$. $\square$

Appendix B - Correlation of incompetent analysts' signals

In this Appendix, I consider the case where the signals of incompetent analysts are (conditionally) correlated rather than those of competent analysts' signals. I show that the main results of the paper remain qualitatively the same. Reputation is not necessarily monotonic in forecast accuracy; a competent analyst is better off when he covers the same firm as the established analyst, and vice versa for an incompetent analyst. While the results are the same, some differences arise, which I emphasize in what follows.

Let us assume that the signals of competent analysts are independent of each other as well as of the signal of incompetent analysts. On the other hand, assume that the signals of two incompetent analysts, say 1 and 2, are conditionally correlated. Let the signals of the two incompetent analysts be s_1^j and $s_2^{j'}$, $j, j' \in \{m, v\}$. Then

$$
\begin{aligned}
&Pr(s_1^j = s_2^j|\pi^j) = \rho b + (1-\rho)b^2 \\
&Pr(s_1^j, s_2^j|\pi^{j'}) = \rho(1-b) + (1-\rho)(1-b)^2 && j \neq j' \\
&Pr(s_1^j, s_2^{j'}|\pi^j) = Pr(s_1^{j'}, s_2^j|\pi^j) = (1-\rho)b(1-b) && j \neq j'
\end{aligned}
$$

Where the parameter ρ reflects the level of correlation.

As in Sections 3 and 4, we restrict attention to the case of uniform prior, i.e., $\alpha = \frac{1}{2}$.

For a new analyst who covers firm B, the firm without prior coverage, the ex-post reputation remains the same. On the other hand, the ex-post reputation for a new analyst who covers firm A is given by

$$\tilde{\theta}_n(f_n^m, f_e^m, \pi^m) = \tilde{\theta}_n(f_n^v, f_e^v, \pi^v) = \frac{\theta_n(\theta_e g^2+(1-\theta_e)gb)}{\theta_n(\theta_e g^2+(1-\theta_e)gb)+(1-\theta_n)(\theta_e bg+(1-\theta_e)(\rho b+(1-\rho)b^2))}$$

$$\tilde{\theta}_n(f_n^m, f_e^v, \pi^m) = \tilde{\theta}_n(f_n^v, f_e^m, \pi^v) = \frac{\theta_n(\theta_e g(1-g)+(1-\theta_e)g(1-b))}{\theta_n(\theta_e g(1-g)+(1-\theta_e)g(1-b))+(1-\theta_n)(\theta_e b(1-g)+(1-\theta_e)(1-\rho)b(1-b))}$$

$$\tilde{\theta}_n(f_n^m, f_e^m, \pi^v) = \tilde{\theta}_n(f_n^v, f_e^v, \pi^m) =$$

$$= \frac{\theta_n(\theta_e(1-g)^2+(1-\theta_e)(1-g)(1-b))}{\theta_n(\theta_e(1-g)^2+(1-\theta_e)(1-g)(1-b))+(1-\theta_n)(\theta_e(1-b)(1-g)+(1-\theta_e)(\rho(1-b)+(1-\rho)(1-b)^2))}$$

$$\tilde{\theta}_n(f_n^m, f_e^v, \pi^v) = \tilde{\theta}_n(f_n^v, f_e^m, \pi^m) = \frac{\theta_n(\theta_e g(1-g)+(1-\theta_e)(1-g)b)}{\theta_n(\theta_e g(1-g)+(1-\theta_e)(1-g)b)+(1-\theta_n)(\theta_e(1-b)g+(1-\theta_e)(1-\rho)b(1-b))}$$

I begin by showing that a pure strategy equilibrium exists, when analysts issue forecasts according to their signals; Clearly, this is also the most informative equilibrium.

Lemma 2. *In equilibrium, analysts issue forecasts according to their signals. That is*

$$f_i(s_i) = \begin{cases} f_i^v & if \quad s_i = s_i^v \\ f_i^m & if \quad s_i = s_i^m \end{cases}$$

Proof. For an analyst who covers firm B the proof is the same as in Lemma 1.

For an analyst who covers firm A the expected reputation is

$$\mathbb{E}[\tilde{\theta}_n(f_n, f_e, \pi)|s_n] =$$
$$= Pr(\pi^m|s_n)\big(Pr(s_e^m|s_n, \pi^m)\tilde{\theta}_n(f_n, f_e^m, \pi^m)Pr(s_e^v|s_n, \pi^m)\tilde{\theta}_n(f_n, f_e^v, \pi^m)\big)$$
$$+Pr(\pi^v|s_n)\big(Pr(s_e^m|s_n, \pi^v)\tilde{\theta}_n(f_n, f_e^m, \pi^v) + Pr(s_e^v|s_n, \pi^v)\tilde{\theta}_n(f_n, f_e^v, \pi^v)\big)$$

Where $Pr(\pi^j|s_n)$ remains as in equation (1).

The analyst's ex-post reputation is higher when his forecast is accurate, while the established analyst's forecast is inaccurate compared to the opposite case. Formally,

$$\tilde{\theta}_n(f_n^{j'}, f_e^j, \pi^j) < \tilde{\theta}_n(f_n^j, f_e^{j'}, \pi^j)$$

Also, his reputation is higher when both analysts issue accurate forecasts than when both issue inaccurate ones. Formally,

$$\tilde{\theta}_n(f_n^{j'}, f_e^{j'}, \pi^j) < \tilde{\theta}_n(f_n^j, f_e^j, \pi^j)$$

As before, $Pr(\pi^{j'}|s_j) < Pr(\pi^j|s_j)$. Thus, an analyst is better off issuing the forecast that is more likely to be accurate than the other possible forecast. In other words, in equilibrium, analysts issue forecasts according to their signals. □

In the case of conditional correlation of incompetent analysts' signals, there are a few differences in analysts' reputation that should be noted.

Observation 1.

1. $\tilde{\theta}_n(f_n^j, f_e^j, \pi^j) < \tilde{\theta}_n(f_n^j, f_e^{j'}, \pi^j)$

2. $\tilde{\theta}_n(f_n^{j'}, f_e^{j'}, \pi^j) < \tilde{\theta}_n(f_n^{j'}, f_e^j, \pi^j)$

When the signals of incompetent analysts are conditionally correlated, issuing the same forecast as the other analyst is perceived somewhat negatively compared to the case where conditional correlation is between competent analysts' information. Thus, an analyst is better off when he issues an accurate forecast while his peer's forecast is inaccurate compared to the case where both analysts issue accurate forecasts. In a similar vein, an analyst is better off when he issues an inaccurate forecast while the other issues an accurate one than when both issue inaccurate forecasts.

The correlation of information can also result in reputation non-monotonicity.

Proposition 6. *When the signals of incompetent analysts are correlated, and the new analyst covers the same firm as the established one, his reputation is not monotonic in his forecast accuracy if the correlation is sufficiently high, i.e.,* $\overline{\rho} < \rho$*, and the established* *analyst is likely to be incompetent, i.e.,* $\theta_e < \overline{\theta}$.

Where $\overline{\theta} = \frac{b(1-g)}{g(g-b)+b(1-g)}$ *and* $\overline{\rho} = \frac{(g-b)(\theta_e g+(1-\theta_e)b)}{(1-\theta_e)b(1-b)}$.

Proof. The new analyst's reputation is non-monotonic in his forecast accuracy if $\tilde{\theta}_n(f_n^j, f_e^j, \pi^j) < \tilde{\theta}_n(f_n^{j'}, f_e^j, \pi^j)$. That is, when

$$\frac{\theta_n(\theta_e g^2+(1-\theta_e)gb)}{\theta_n(\theta_e g^2+(1-\theta_e)gb)+(1-\theta_n)(\theta_e bg+(1-\theta_e)(\rho b+(1-\rho)b^2))} <$$
$$\frac{\theta_n(\theta_e g(1-g)+(1-\theta_e)(1-g)b)}{\theta_n(\theta_e g(1-g)+(1-\theta_e)(1-g)b)+(1-\theta_n)(\theta_e(1-b)g+(1-\theta_e)(1-\rho)b(1-b))}$$

Simplifying the above inequality, we observe that it holds if and only if $\theta_e < \frac{b(1-g)}{g(g-b)+b(1-g)}$ and $\frac{(g-b)(\theta_e g+(1-\theta_e)b)}{(1-\theta_e)b(1-b)} < \rho$. □

The next immediate question is whether analysts of different abilities differ in their coverage preferences. One might think that because conditional correlation exists only for signals of incompetent analysts, a competent analyst may be better off when covering the firm without prior coverage, i.e., firm B, in contrast to the case where correlation is between competent analysts, where a competent analyst is better off covering the same firm as the established one, and vice versa for an incompetent analyst. Nevertheless, the next proposition suggests that this logic is invalid. That is, a competent analyst expects a better reputation when covering the same firm as the established one compared to covering a firm exclusively, and vice versa for an incompetent analyst.

Proposition 7.

$$\mathbb{E}[\tilde{\theta}_{n,B}(f_{n,B}, \pi_B)|n \textit{ is good}] < \mathbb{E}[\tilde{\theta}_{n,A}(f_{n,A}, f_{e,A}, \pi_A)|n \textit{ is good}]$$
$$\mathbb{E}[\tilde{\theta}_{n,A}(f_{n,A}, f_{e,A}, \pi_A)|n \textit{ is bad}] < \mathbb{E}[\tilde{\theta}_{n,B}(f_{n,B}, \pi_B)|n \textit{ is bad}]$$

Proof.

$$\mathbb{E}[\tilde{\theta}_{n,B}(f_{n,B},\pi_B)|\ n\text{'s type}] =$$
$$Pr(\pi^m|s_n^j,\ n\text{'s type})\tilde{\theta}_n(f_n^j,\pi^m) + Pr(\pi^v|s_n^j,\ n\text{'s type})\tilde{\theta}_n(f_n^j,\pi^v)$$

and

$$\mathbb{E}[\tilde{\theta}_{n,A}(f_{n,A},f_{e,A},\pi_A)|\ n\text{'s type}] = Pr(\pi^m|s_n^j,\ n\text{'s type})\mathbb{E}_{s_e|\pi^m}[\tilde{\theta}_n(f_n,f_e(s_e),\pi^m)]$$
$$+Pr(\pi^v|s_n^j,\ n\text{'s type})\mathbb{E}_{s_e|\pi^v}[\tilde{\theta}_n(f_n,f_e(s_e),\pi^v)]$$

where

$$\mathbb{E}_{s_e|\pi^j,s_n}[\tilde{\theta}_n(f_n,f_e(s_e),\pi^j)] =$$
$$Pr(s_e^j|\pi^j,s_n)\tilde{\theta}_n(f_n,f_e^j,\pi^j) + Pr(s_e^{j'}|\pi^j,s_n)\tilde{\theta}_n(f_n,f_e^{j'},\pi^j)$$

Now, $Pr(\pi^m|s_n^j,\ n\text{'s type})$ is given in equation (1), and the set of ex-post reputation is given above.

As before differences in an analyst's expected reputation can arise only from the probabilities $Pr(s_e^j|\pi^j,s_n)$ and $Pr(s_e^{j'}|\pi^j,s_n)$.

Let us assume, w.l.o.g., that the new analyst observes the signal s_n^m.

Since now the correlation is between the signals of incompetent analysts, they are the ones who expect to observe similar signals. Hence, from an incompetent analyst's point of view

$$Pr(s_e^m|\pi^m,s_n^m) = eg + (1-e)(\rho + (1-\rho)b$$

and

$$Pr(s_e^v|\pi^m, s_n^m) = e(1-g) + (1-e)(1-\rho)(1-b)$$

However, for a competent analyst

$$Pr(s_e^m|\pi^m, s_n^m) = eg + (1-e)b$$

and

$$Pr(s_e^v|\pi^m, s_n^m) = e(1-g) + (1-e)(1-b)$$

Moreover, using the ex-post reputation derived above, we observe that

$$\tilde{\theta}_n(f_n^m, \pi^m) < (eg+(1-e)b)\tilde{\theta}_n(f_n^m, f_e^m, \pi^m) + (e(1-g)+(1-e)(1-b))\tilde{\theta}_n(f_n^m, f_e^v, \pi^m)$$

and

$$\begin{aligned} &\tilde{\theta}_n(f_n^v, \pi^m) < \\ &(eg + (1-e)b)\tilde{\theta}_n(f_n^m, f_e^v, \pi^v) + (e(1-g) + (1-e)(1-b))\tilde{\theta}_n(f_n^m, f_e^m, \pi^v) \\ &\qquad < \tilde{\theta}_n(f_n^v, \pi^m) \end{aligned}$$

Thus, a competent analyst is better off when he covers firm A.

In contrast,

$$\begin{aligned} &(eg + (1-e)(\rho + (1-\rho)b))\tilde{\theta}_n(f_n^m, f_e^m, \pi^m) \\ &\quad +(e(1-g) + (1-e)(1-\rho)(1-b))\tilde{\theta}_n(f_n^m, f_e^v, \pi^m) < \tilde{\theta}_n(f_n^m, \pi^m) \end{aligned}$$

and

$$
\begin{aligned}
&(eg + (1-e)(1-\rho)b)\tilde{\theta}_n(f_n^m, f_e^v, \pi^v) \\
&\quad + (e(1-g) + (1-e)(\rho + (1-\rho)(1-b))\tilde{\theta}_n(f_n^m, f_e^m, \pi^v) < \tilde{\theta}_n(f_n^v, \pi^m)
\end{aligned}
$$

Thus, an incompetent analyst is better off when covering firm B. □

The reason that an incompetent analyst is worse off when he covers firm A is that he expects to issue the same forecast as the established analyst with more significant probability than a competent analyst. In contrast, a competent analyst is more likely to be accurate when the established analyst issues an inaccurate forecast, and thus, expects to gain a better reputation when covering firm A.

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CHAPTER 2

Voluntary Disclosure of Firms Covered by Analysts with Uncertain Forecasting Objective

co-authored with Beatrice Michaeli

1 Introduction

Corporate disclosures are the primary but not exclusive source of information in financial markets. Over the past several years, additional sources of information, such as sell-side analysts, have captured the attention of researchers and regulators. This is not surprising given that, in addition to directly mitigating information asymmetry in the marketplace, these sources may have a significant impact on corporate disclosures. For example, firms tend to provide favorable information, such as expected high market demand, while concealing unfavorable information, such as expected low demand.[17] In their forecasts, analysts covering the same industry also provide relevant information to the market participants. The economic incentives of analysts have long been debated in the literature. Many analysts aim to issue accurate forecasts and achieve an All-star analyst status.[18] There is also empirical and anecdotal evidence that some analysts provide biased forecasts due to various economic incentives such as currying favor for firms or generating trading commissions.[19] It is unclear whether the possibly biased informa-

[17]Because of their uncertainty about what the firm knows, investors cannot unravel the firm's private information (Dye 1985).

[18]See for example Mikhail, Walther, and Willis (1999), Hong, Lim, and Stein (2000).

[19]Evidence consistent with analysts maximizing trading volume to earn trading commissions has been provided in Ljungqvist, Marston and Wilhelm (2006) and Karmaziene (2020). Related, Cowen,

tion provided by analysts in their forecasts encourages or curbs the firm's tendency to disclose its private information. In this paper, we analyze the effect of analyst coverage with uncertain forecasting objective on corporate disclosures and market prices.

We consider a model where a firm ("it"), which may be endowed with information about the product demand, decides whether to voluntarily (and truthfully) disclose this information to investors ("they"). The firm maximizes the market price, which reflects the investors' expectations regarding the product demand in the market. The focus of our study is the effect of analysts' coverage—as an additional source of information available to investors—on the firm's tendency to disclose its private information.[20] We assume that an analyst ("she") covers the firm's industry and always observes information about the market demand.[21] Whether the analyst biases her forecast or issues, in an accurate manner, her information about the market demand is only privately known to the analyst. The investors, as well as the firm, are uncertain about the analyst's forecasting objective. We consider scenarios where the firm's disclosure precedes or succeeds the issuance of the analyst's forecast and illustrate the existence of a unique threshold equilibrium, where the firm discloses favorable news and withholds unfavorable ones.[22]

The firm's information endowment is assumed to be independent of the analyst's

Groysberg and Healy (2006), Jacob, Rock and Weber(2008), Agrawal and Chen (2007), and Clarke, Khorana, Patel and Rau (2006) report that investment bank analysts tend to issue pessimistic forecasts. In contrast, Brown, Foster and Noreen (1985), O'Brien (1988), Womack (1996), McNichols and O'Brien (1997) and Hong and Kubik (2003) provide evidence consistent with positive bias. For a detailed survey of the analysts' literature, see Ramnath, Rock and Shane (2006). Schipper (1991) calls for more research on analysts' economic incentives.

[20] In addition to corporate disclosures, our model can be applied to a variety of contexts where a party's tendency to communicate private information may be affected by external sources of information. For example, the model can be applied to a case of a politician disclosing information that voters may also obtain from the media or political opponents.

[21] There is no inconsistency in assuming that the firm obtains information with some probability, whereas the analyst always obtains information about an event occurring outside of the firm, e.g., future demand for products.

[22] As in Dye (1985), unraveling does not occur because of the uncertainty about the firm's information endowment.

forecasting objective. Nevertheless, we demonstrate that the investors' beliefs about these uncertain events are *endogenously intertwined* and *reinforce* each other. For example, when investors believe that the firm is informed, they believe that it is more likely that the analyst is unbiased compared to the case where the firm is believed to be uninformed. Thus, the analyst affects the market price not only through the information she provides in her forecast (the direct effect) but also through the effect her forecast has on investors' beliefs regarding the firm's information endowment (the indirect effect), which, in turn, affects the firm's disclosure decision.

We start with the case where the analyst issues the relevant forecast before the firm can disclose information to the market. This situation may arise, for example, when analysts provide their assessments about the industry they cover before the firm's conference call, where the firm's management has the opportunity to communicate information to investors. In this case, the firm observes the analyst's forecast and may respond by disclosing its assessment of the future demand. In equilibrium, for any forecast issued by the analyst, the firm would respond only if its information exceeds some threshold (that depends on the analyst's forecast). We show that the firm may respond even to positively biased forecasts, i.e., forecasts that assess higher demand than can be indicated from the firm's information. The intuition for this disclosure of (seemingly) bad news is the uncertainty that the investors have regarding the analyst's forecasting objective. The investors are skeptical about the analyst's forecast, and thus, a positively biased forecast may lead to a market price that is below the one the firm can achieve by disclosing information.

We proceed with the case where the firm's possible disclosure precedes (or coincides) with the analyst's forecast. Since the firm does not observe the analyst's forecast when making its disclosure decision, it must form an expectation about the forecast. We show that the firm may benefit from relying on the analyst to forecast the same information that the firm observed in a potentially biased way. This benefit arises from the "reinforcement role" of forecasts on investors' beliefs, and the investors' uncertainty about the analyst's objective and the firm's information endowment. Although we assume that the firm's information endowment is independent of the analyst's forecasting objective, the investors' beliefs about these events are *endogenously intertwined*, and *reinforce each other*. In particular, a product demand forecasted by the analyst that is below the disclosure threshold strengthens the investors' beliefs that the analyst is unbiased, which in turn, reinforces investors' beliefs that the firm is informed but did not disclose. And vice versa: a forecast above the disclosure threshold is perceived to be issued with higher probability by a biased analyst than by an unbiased one. Therefore, such a forecast strengthens the investors' beliefs that the firm is uninformed. Put differently, the analyst's forecast "reinforces" investors' beliefs about the firm's information endowment—if the investors believe that the forecast is biased (unbiased), they also believe that the firm is less (more) likely to be informed. The reinforcement role of (potentially biased) forecasts is at the heart of our main results. We find that analyst coverage succeeding corporate disclosures has a crowding-out effect in that a firm facing coverage is less likely to disclose its information.

We also consider the optimal disclosure timing of the firm and show that firms observing high product demand prefer to delay their disclosure until after the analyst forecast. The reason is that by delaying disclosure they can only benefit from potential nondisclosure at a later date. In contrast, firms observing unfavorable market demand may benefit from credibly committing to nondisclosure prior to the analyst forecast (if they could). Lastly, we extend our results to a setting in which the firm's disclosure timing is fixed, but the analyst can choose the timing at which she issues the forecast. We illustrate that biased analysts may prefer to forecast early to preempt the firm's disclosure.

Broadly speaking, our paper belongs to the analytical literature on voluntary disclosure initiated by Grossman (1981) and Milgrom (1981) and surveyed by Beyer, Cohen, Lys, and Walther (2010). Similar to the analytical models of Dye (1985), Farrel and Sobel (1983), and Jung and Kwon (1988), we allow for firms to be endowed with information about the state with some probability. However, unlike these papers, we consider additional sources of information, in particular, a sell-side analyst who issues a forecast that affects the firm's price. Studying the effect this forecast has on voluntary disclosure, and the dynamic of interaction between the two means of information transmissions is our main contribution to the literature on voluntary disclosure.

The effect of external sources of information on voluntary disclosure is also the focus of several recent papers. Frenkel, Guttman, and Kremer (2019) also consider a disclosure problem where a firm faces analyst coverage.[23] In a model where the probability that an analyst observes information depends on the probability that the firm is endowed with information, they find that analyst coverage can crowd in or crowd out voluntary disclosures.[24] In contrast, we assume that the analyst always observes information but may bias the information she forecasts. In our model, the probability that the firm is endowed with information is independent of the probability that the forecast is unbiased. Nevertheless, the investors' beliefs about these events endogenously intertwined, and we find that analyst coverage suppresses voluntary disclosure. In addition to studying the firm's disclosure decision, Frenkel, Guttman, and Kremer (2019) study stock price efficiency and liquidity, whereas we focus on the dynamic interaction between voluntary disclosure and analyst coverage.[25] Our paper is also related to Einhorn (2018), who analyzes voluntary disclosures in the presence of competing information

[23]In a related model, Ebert, Schäfer, and Schneider (2019) study different types of information leaks.

[24]Dye and Sridhar (1995) consider a model with multiple firms where disclosure by one of the firms causes an update in the investors' beliefs about the information endowment of the other firms.

[25]In a model with a firm that cares about stock prices in two periods, Guttman, Kremer, and Skrzypacz (2014) find that disclosing in the second period is interpreted more favorably.

sources and explains a deviation from full disclosure equilibrium to one with partial and selective disclosure.

Our analysis of disclosures preceding analysts' forecasts is related to the literature on uncertain investors' reactions (e.g., Dutta and Trueman 2002 and Suijs 2007). While this literature focuses on the uncertainty pertaining to how investors react to disclosure, in our model, the uncertainty pertains to how investors react to nondisclosure. Our analysis of disclosure succeeding analysts' forecasts is related to the literature studying shifts in the investors' pre-disclosure expectations of the state (Jung and Kwon 1988, Acharya, DeMarzo and Kremer 2011). In addition to affecting the investors' prior expectation of future demand, forecasts in our model also affect the investors' beliefs about the analyst's forecasting objective and the firm's information endowment.

The paper proceeds as follows. Section 2 describes the economic setting and considers as a benchmark the voluntary disclosure in the absence of analyst coverage. Section 3 discusses the market price and the reinforcement effect of analyst's forecasts. Section 4 considers the firm's voluntary disclosure that succeeds or precedes the analyst forecast. Section 5 studies the optimal timing of corporate disclosures and analyst's forecasts. Section 6 discusses empirical implications. Section 7 concludes.

2 Economic setting and benchmark

2.1 Setting

The model entails a firm ("it") facing uncertain product demand. The payoff from high demand is normalized to one, and the payoff from low demand is normalized to zero. We assume that the likelihood of high demand, $\theta \in [0, 1]$, is distributed according to a probability distribution function $g(\cdot)$ and a cumulative distribution function $G(\cdot)$, with a prior expectation $\mathbb{E}[\theta] = \mu$. The firm is informed about θ with probability

$\Pr(\mathit{Inf}) = p \in (0,1)$ and is uninformed with probability $\Pr(\mathit{NoInf}) = 1 - p \in (0,1)$. Here, $\mathcal{I} = \{\mathit{Inf}, \mathit{NoInf}\}$ denotes the firm's information endowment, which we assume is independent of θ. If the firm is informed about the value, it may voluntarily disclose it to investors ("they") at no cost. Following the voluntary disclosure literature, we assume that any disclosed value is verifiable and thus truthful. If the firm is uninformed, it cannot credibly communicate the lack of information endowment. For future reference, $d \in \{\theta, \emptyset\}$ denotes the firm's disclosure ($d = \theta$) or lack of disclosure ($d = \emptyset$).

An analyst ("she") covers the industry in which the firm operates and therefore always observes information about the market demand.[26] After observing θ, the analyst releases a forecast f. The forecasting objective of the analyst is uncertain in the sense that, with probability $q \in (0,1)$, she is unbiased, maximizes a payoff given by:

$$\pi_U = -(f - \theta)^2 \tag{7}$$

and issues an unbiased forecast that reflects the true state, $f = \theta$.[27] Otherwise, the analyst is biased, maximizes a payoff given by:

$$\pi_B = -(f - \beta)^2 \tag{8}$$

and issues a forecast $f = \beta$, where $\beta \in [0,1]$ is the value towards which the analyst is biased. The payoff function in (8) could represent for example the desire of the analyst to generate trading volume or may reflect an inherent personality trait.[28] The value β

[26]For tractability reasons we assume that the analyst's information is precise. Our results will hold qualitatively if the analyst observes θ with noise.

[27]If $q = 0$, the forecast is uninformative and must be ignored by the investors. On the other hand, if $q = 1$, the forecast fully reveals the state, and thus, the firm's disclosure decision is irrelevant. Hence we focus on the interesting case of $q \in (0,1)$.

[28]The payoff function in (8) is a parsimonious way to model the analyst's forecasting bias. The analyst's objective could be to minimize the difference between the market price and a specific unknown value. As we show in the next section, the market price is (weakly) increasing in the forecast, and therefore a payoff minimizing the difference between price and a certain value is analogous to the payoff

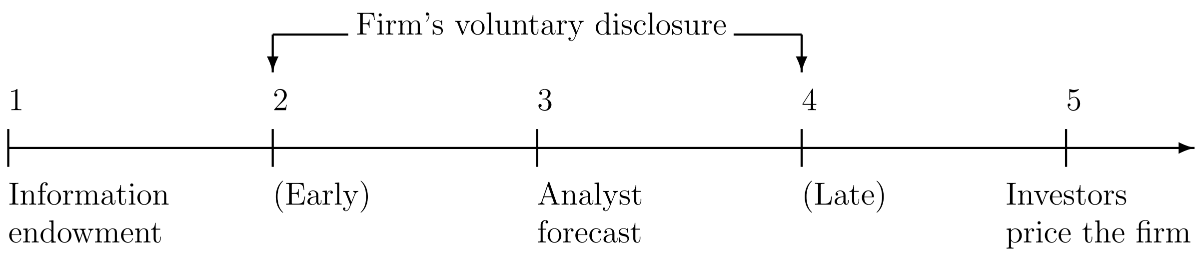

Figure 1: Timeline of events

is unknown to the firm and the investors.[29] It is distributed according to a probability distribution function $h(\cdot)$ and a cumulative distribution function $H(\cdot)$. Let $i = \{U, B\}$ denote the analyst's type. We refer to an analyst of type $i = U$ as "unbiased" and to an analyst of type $i = B$ as "biased."[30]

After observing the analyst's forecast and the firm's disclosure, the investors price the firm at its expected value,

$$P(d, f) = \mathbb{E}[\theta \times 1 + (1 - \theta) \times 0 | d, f] = \mathbb{E}[\theta | d, f].$$

As standard for disclosure models, the firm maximizes $P(d, f)$.

The timeline of events is illustrated in Figure 1. At date 1, the analyst observes θ. The firm observes θ with probability p. At date 3, the analyst issues a forecast. The firm decides whether to disclose θ either at date 2 (under the "early disclosure" scenario), or at date 4 (under the "late disclosure" scenario). At date 5, the investors

in (8).

[29]The value towards the analyst is biased can change over time. Thus the investors may not learn β even in a multi-period setting.

[30]Our model can be applied to other contexts beyond one of corporate disclosures made in the presence of analyst coverage. For example, it is descriptive of the incentives of a politician to voluntarily disclose information about a political situation to a group of voters who decide about their support of the politician's agenda based on the voluntary disclosure and the information provided by a journalist with an uncertain objective.

price the firm. We first consider the timing of the firm's disclosure as exogenously given. That is, in Section 4, the firm either faces a late disclosure scenario (Section 4.1) or an early disclosure scenario (Section 4.2), potentially because of the date at which the firm observes its information or for other exogenous reason. In Section 5.1, we consider the firm's optimal choice of disclosure timing. In Section 5.2, we take a different approach of considering the problem: we take the timing of the firm's disclosure as fixed and discuss the analyst's optimal timing for forecast release.

2.2 Benchmark

Throughout the paper, we refer to the case without analyst coverage as the benchmark case.[31] The superscript "$\mathcal{D}$" denotes the benchmark case as studied in Dye (1985).

Lemma 0. [**Dye 1985 and Jung and Kwon 1988**] *When $q = 0$, there exists a unique threshold $\theta^{\mathcal{D}} \in (0, \mu)$, such that the firm discloses if $\theta \geq \theta^{\mathcal{D}}$ and withholds otherwise. The threshold $\theta^{\mathcal{D}}$ is decreasing in the probability of information endowment, p.*

From a technical perspective, the unique threshold equilibrium arises because, while the market price following disclosure is equal to the state θ (and thus increasing at a rate of 1 with the state), the price following nondisclosure is constant and always between zero and the prior mean μ.

The key insight of the Dye model is that firms are able to withhold information and pretend to be uninformed because the market price following nondisclosure depends on the investors' beliefs about the firm's information endowment. In this paper, we show that coverage of analysts with uncertain forecasting objective affects the ability of firms to pretend to be uninformed in two ways. First, because the forecast represents an additional signal of the state, it directly affects the market price and thereby the

[31] In our setting, this case is equivalent to setting $q = 0$ so that the analyst is always biased and her forecast will be ignored by the investors.

firm's disclosure decision. We dub this the "direct effect." Second, the forecast affects the investors' beliefs about the analyst's objective and their beliefs about the firm's information endowment. As we show below, these beliefs are endogenously intertwined. As a result, the forecast indirectly affects the market price and thereby the firm's disclosure decision. We dub this the "indirect effect."

3 Market price and reinforcement of beliefs

We solve the model by backward induction. At date 5, after observing the firm's disclosure and the analyst's forecast, the investors price the firm. When the firm discloses, the market price is determined only by the disclosed value, regardless of the analyst's forecast, i.e.,

$$P(\theta, f) = \mathbb{E}[\theta|\theta, f] = \theta. \tag{9}$$

In this case, the investors disregard the forecast because the firm learns the state precisely and, if it decides to disclose, it does so truthfully.[32] When the firm remains silent, the investors observe the lack of disclosure, $d = \emptyset$, and the forecast, f. From the investors' point of view, the forecast equals the state θ with probability q and equals some unknown value β with probability $1 - q$. Hence, in contrast to the benchmark case, now the nondisclosure price is not constant but is increasing in the state θ. As a result, it is not immediate that a threshold equilibrium even exists. In Section 4, we establish the existence of such equilibrium under both (late and early) disclosure scenarios. For now, we assume that a threshold equilibrium indeed exists, with some disclosure threshold $\widehat{\theta} \in [0, 1]$.[33]

[32] While the analyst also learns the state precisely, she may be biased and issue a forecast of $f = \beta$.

[33] In this section, we use the notation $\widehat{\theta}$ for a generic threshold. In the following sections, we denote the threshold in the late disclosure scenario $\theta^{\mathcal{L}}$ and the threshold in the early disclosure scenario $\theta^{\mathcal{E}}$.

When the investors determine the firm's price, they consider four possible events:

(1) With probability $\Pr(\mathit{NoInf} \cap i = U | \emptyset, \widehat{\theta}, f)$, the firm is uninformed and the analyst is unbiased. When the analyst is unbiased, her forecast reflects the true state. Thus the investors' expectation of the state in this case is simply the forecast, $\mathbb{E}[\theta | f = \theta] = f$.

(2) With probability $\Pr(\mathit{NoInf} \cap i = B | \emptyset, \widehat{\theta}, f)$, the firm is uninformed and the analyst is biased. Because there is nothing to learn from the firm's nondisclosure and from the analyst's forecast, the investors' expectation of the state is just the prior, $\mathbb{E}[\theta] = \mu$.

(3) With probability $\Pr(\mathit{Inf} \cap i = U | \emptyset, \widehat{\theta}, f)$, the firm is informed and the analyst is unbiased. As in case (1), because the analyst is unbiased, her forecast reflects the true state. Thus the investors' expectation of the state is $\mathbb{E}[\theta | f = \theta] = f$.

(4) With probability $\Pr(\mathit{Inf} \cap i = B | \emptyset, \widehat{\theta}, f)$, the firm is informed and the analyst is biased. When the analyst is biased, there is nothing to learn from her forecast. However, because the investors believe that the firm is informed, they also believe that the firm observed $\theta < \widehat{\theta}$. Thus the investors' expectation is $\mathbb{E}[\theta | \theta < \widehat{\theta}]$.

To reduce clutter, let $\Lambda(\mathcal{I}, i | \widehat{\theta}, f) \equiv \Pr(\mathcal{I} \cap i | \emptyset, \widehat{\theta}, f)$, represent the joint probability that the firm's information endowment is $\mathcal{I} = \{\mathit{Inf}, \mathit{NoInf}\}$ and the analyst's type is $i = \{U, B\}$, conditional on nondisclosure $d = \emptyset$, threshold $\widehat{\theta}$ and forecast f. Summarizing the preceding discussion, the market price in case of nondisclosure is given by:

$$\begin{aligned} P(\emptyset, f | \widehat{\theta}) \;\; = \;\; & \Big(\Lambda(\mathit{NoInf}, U | \widehat{\theta}, f) + \Lambda(\mathit{Inf}, U | \widehat{\theta}, f) \Big) \times f \\ & + \Lambda(\mathit{NoInf}, B | \widehat{\theta}, f) \times \mu + \Lambda(\mathit{Inf}, B | \widehat{\theta}, f) \times \mathbb{E}[\theta | \theta < \widehat{\theta}]. \end{aligned} \tag{10}$$

It is apparent that the nondisclosure price in (10) is a weighted average of the analyst's forecast f, the prior μ, and the posterior $\mathbb{E}[\theta|\theta < \widehat{\theta}]$. For comparison, the nondisclosure price in the benchmark case is only a weighted average of the prior expectation and the posterior expectation of the state, conditional on being below the disclosure threshold (Dye 1985). However, as we show in detail below, the effect of an analyst with uncertain forecasting objective goes beyond a mere addition of yet another information source. Even though the type of the analyst is independent of the firm's information endowment, the beliefs of the investors about the endowment and the analyst's type are *endogenously intertwined* and *reinforce* each other.

To see how, let $\gamma(\widehat{\theta}) \equiv \Pr(\mathit{NoInf}|\emptyset, \widehat{\theta})$ be the probability that the firm is uninformed in the absence of analyst coverage. Consider the two events involving the firm being uninformed, $\mathcal{I} = \mathit{NoInf} \cap i = U$ and $\mathcal{I} = \mathit{NoInf} \cap i = B$. If the investors believe that the firm is uninformed, then the observed forecast carries no information about the analyst's type. Thus, $\Lambda(\mathit{NoInf}, U|\widehat{\theta}, f) = \Pr(i = U)\Pr(\mathit{NoInf}|\emptyset, \widehat{\theta}) = q \times \gamma(\widehat{\theta})$ for any $f \in [0,1]$. Similarly, $\Lambda(\mathit{NoInf}, B|\widehat{\theta}, f) = \Pr(i = B)\Pr(\mathit{NoInf}|\emptyset, \widehat{\theta}) = (1-q) \times \gamma(\widehat{\theta})$. In summary, the probability that the event $\mathcal{I} = \mathit{NoInf} \cap i = U$ and the probability that the event $\mathcal{I} = \mathit{NoInf} \cap i = B$ occur are independent of the analyst's forecast.

When the investors believe that the firm is informed, they also infer that the firm observed $\theta < \widehat{\theta}$ and choose to remain silent. Therefore the observed forecast carries information about the analyst's type. Thus the probability that the event $\mathcal{I} = \mathit{Inf} \cap i = U$ and the probability that the event $\mathcal{I} = \mathit{Inf} \cap i = B$ occurs crucially depends on the analyst's forecast. When the investors observe a forecast that exceeds the disclosure threshold, it will be rationally inconsistent for them to believe that the firm is informed and the analyst is unbiased. Specifically, the probability that the analyst is unbiased, conditional on the the firm being informed and $f > \widehat{\theta}$ is zero. Thus

$$\begin{aligned}\Lambda(\mathit{Inf}, U|\widehat{\theta}, f > \widehat{\theta}) &= \Pr(i = U|\mathit{Inf}, \emptyset, \widehat{\theta}, f > \widehat{\theta}) \Pr(\mathit{Inf}|\emptyset, \widehat{\theta}, f > \widehat{\theta}) \\ &= 0 \times \Pr(\mathit{NoInf}|\emptyset, \widehat{\theta}) \\ &= 0.\end{aligned}$$

On the other hand, the probability that the analyst is biased, conditional on the firm being informed and the forecast exceeding the disclosure threshold is one. As a result, $\Lambda(\mathit{Inf}, B|\widehat{\theta}, f > \widehat{\theta}) = \Pr(\mathit{Inf}|\emptyset, \widehat{\theta}) = 1 - \gamma(\widehat{\theta})$

Let us now consider the case where the analyst's forecast is below the disclosure threshold. Then, it is possible that the analyst is unbiased and the firm is informed. Specifically, when the firm is informed and remains silent, the forecast is below the disclosure threshold either because the analyst is unbiased or because the analyst is biased but it just happened that her forecast is below the disclosure threshold. As a result,

$$\begin{aligned}\Lambda(\mathit{Inf}, U|\widehat{\theta}, f < \widehat{\theta}) &= \Pr(i = U|\mathit{Inf}, \emptyset, \widehat{\theta}, f < \widehat{\theta}) \Pr(\mathit{Inf}|\emptyset, \widehat{\theta}, f < \widehat{\theta}) \\ &= \delta(\widehat{\theta}) \times (1 - \gamma(\widehat{\theta})),\end{aligned}$$

where $\delta(\widehat{\theta}) \equiv \Pr(i = U|\mathit{Inf}, \emptyset, f < \widehat{\theta}) = \frac{q}{q+(1-q)H(\widehat{\theta})} \in [q, 1]$ is the probability that the analyst is unbiased, conditional on the firm being informed and remaining silent, and the forecast being below the threshold $\widehat{\theta} \in [0, 1]$. Similarly, $\Lambda(\mathit{Inf}, B|\widehat{\theta}, f < \widehat{\theta}) = (1-\delta(\widehat{\theta})) \times (1-\gamma(\widehat{\theta}))$. Comparing the beliefs of the investors yields the following lemma.

Lemma 1. *Lower forecasts strengthen (weaken) the joint beliefs of the investors that the firm is informed and the analyst is unbiased (biased) in the sense that:* $\Lambda(\mathrm{Inf}, U|\widehat{\theta}, f) = \mathbb{1}_{f \le \widehat{\theta}} \times \delta(\widehat{\theta}) \times (1 - \gamma(\widehat{\theta})) + (1 - \mathbb{1}_{f \le \widehat{\theta}}) \times 0$ *and* $\Lambda(\mathrm{Inf}, B|\widehat{\theta}, f) = \mathbb{1}_{f \le \widehat{\theta}} \times (1 - \delta(\widehat{\theta})) \times (1 -$

$\gamma(\widehat{\theta})) + (1 - \mathbb{1}_{f \leq \widehat{\theta}}) \times (1 - \gamma(\widehat{\theta}))$.

Going back to the nondisclosure price in (10), and comparing it with the benchmark case, we note that analyst coverage with uncertain forecasting objective not only shifts weight from the prior μ and the posterior $\mathbb{E}[\theta|\theta \leq \widehat{\theta}]$ to the forecast f but also redistributes the relative weights on μ and $\mathbb{E}[\theta|\theta \leq \widehat{\theta}]$. To illustrate this effect suppose that the forecast is below the disclosure threshold, i.e., $f \leq \widehat{\theta}$. This is a case in which the investors cannot rule out that the analyst is unbiased, regardless of their beliefs about the firm's information endowment. Then, by Lemma 1, the relative weight on the prior μ is larger than the weight on the posterior $\mathbb{E}[\theta|\theta \leq \widehat{\theta}] < \mu$ (as compared to these weights under the benchmark case). Now suppose that the forecast is above the disclosure threshold. As already noted, to hold consistent beliefs, the investors cannot simultaneously believe the forecast is unbiased and the firm is informed. Then, by Lemma 1, the relative weight on the prior μ is lower than the weight on the posterior $\mathbb{E}[\theta|\theta \leq \widehat{\theta}] < \mu$ (as compared to the benchmark case).

To gain further intuition about the driving forces behind our result, let us focus on the beliefs of the investors that the analyst is unbiased.

Lemma 2. *The investors' beliefs that the analyst is unbiased are* $\Pr(i = U|\mathit{NoInf}, \emptyset, \widehat{\theta}) = q$ *and* $\Pr(i = U|\mathit{Inf}, \emptyset, \widehat{\theta}) = \mathbb{1}_{f \leq \widehat{\theta}} \times \delta(\widehat{\theta}) + (1 - \mathbb{1}_{f \leq \widehat{\theta}}) \times 0$*, where* $\delta(\widehat{\theta}) \geq q$*.*

Our result is graphically illustrated in Figure 2. If the investors believe that the firm is uninformed, then their beliefs regarding the analyst's objective remain at the prior level $\Pr(i = B) = q$, regardless of the forecast. However, if the investors believe that the firm is informed and did not disclose, they infer that the state is below the disclosure threshold (because the investors are aware of the firm's disclosure strategy). Then, if the analyst forecasts a value that is above the disclosure threshold, the investors conclude that the forecast cannot reflect the true state and allocate a probability of

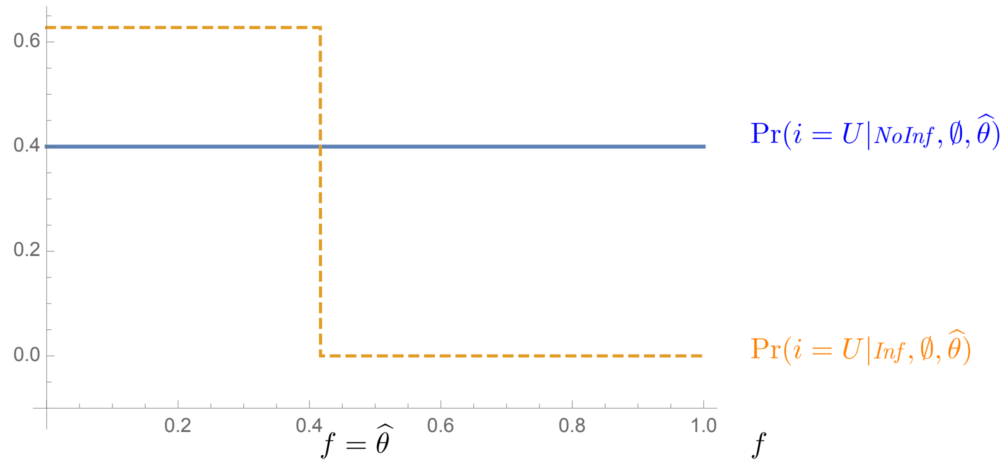

Figure 2: Investors' beliefs that the analyst is unbiased
Numerical example with uniform distributions of θ and β, $p = 0.6$ and $q = 0.4$.

0 that the analyst is unbiased.[34] If the analyst forecasts a value that is below the disclosure threshold, the investors conclude that the probability for the forecast to be unbiased is $\gamma(\widehat{\theta}) > q$. In other words, the investors update upwards their beliefs that the analyst is unbiased. The following corollary follows immediately:

Corollary 2. *Regardless of the value β towards which the analyst may be biased, the investors are always more skeptical about forecasts reflecting high values.*

Regardless of the investors' beliefs about the firm's information endowment, analyst's forecasts above the disclosure threshold are considered to be (weakly) more biased. Surprisingly, this happens even when the analysts are biased towards the lowest possible value $\beta = 0$. This is because the investors' skepticism is not driven by the direction of the analysts' bias, rather by how their forecasts compare with the equilibrium disclosure threshold of the firm.

Another way of thinking about the effect of analysts' forecasts is by describing how the realization of f affects the investors' beliefs about the firm's information endowment,

[34]To hold consistent beliefs, the investors cannot believe that the forecast truthfully reflects a state exceeding the disclosure threshold and that the firm is informed and remained silent.

$\Pr(\textit{NoInf}|\emptyset, \widehat{\theta}, f)$.

Lemma 3. *Forecasts below (above) the disclosure threshold weakens (strengthens) the investors' beliefs that the firm is uninformed, i.e.,* $\Pr(\textit{NoInf}|\emptyset, \widehat{\theta}, f < \theta^{\mathcal{E}}) < \Pr(\textit{NoInf}|\emptyset, \widehat{\theta}) < \Pr(\textit{NoInf}|\emptyset, \widehat{\theta}, f > \theta^{\mathcal{E}})$.

To understand the intuition behind our result, consider the case where the firm remains silent and the analyst forecasts a value that is below the disclosure threshold. Then, the investors believe that it is more likely that the analyst is unbiased and forecasted the true state. This, in turn, reinforces the investors' beliefs that the firm is informed but did not disclose. And vice versa: a forecast above the disclosure threshold is perceived to be more biased. Therefore it strengthens the investors' beliefs that the firm is uninformed. Put differently, the forecast "reinforces" the investors' beliefs about the firm's information endowment—if the investors believe that the analyst is biased (unbiased), they also believe that the firm is less (more) likely to be informed. This effect, which we label the "reinforcement effect" of forecasts, plays a significant role in our analysis.

4 Firm's voluntary disclosure

4.1 Late disclosure

We begin with the case where the analyst releases her forecast before the firm has an opportunity to disclose. In this scenario, the firm, if informed, can respond at date 4 to the analyst's forecast by disclosing the observed value θ ("late disclosure"). We first the existence of a unique threshold equilibrium that depends on the forecast f. The superscript "$\mathcal{L}$" denotes late disclosure.

Proposition 1. *For any forecast $f \in [0,1]$ there exists a threshold $\theta^{\mathcal{L}} \in (0,1)$, such that the informed firm responds if $\theta \geq \theta^{\mathcal{L}}$ and remains silent otherwise.*

Our result suggests that firms observing states above $\theta^{\mathcal{L}}$ "correct" biased forecasts, while the rest remain silent—even if, or perhaps *because*, the forecast does not reflect the true state.[35]

To understand the intuition behind our result, note that the firm compares the price following disclosure, as defined in (9), with the price following nondisclosure, as defined in (10) for a threshold $\widehat{\theta} = \theta^{\mathcal{L}}$. Importantly, for a given forecast f, the price in (10) is constant. In addition, because of the investors' uncertainty regarding the analyst's type and the weight put on the prior expectation, the nondisclosure price is above 0 even when $f = 0$ and below 1 even when $f = 1$. Therefore a unique threshold equilibrium arises such that the price following disclosure of $\theta = \theta^{\mathcal{L}}$ equals the nondisclosure price $P(\emptyset, f|\theta^{\mathcal{L}})$. Put differently, because of their uncertainty about the analyst's forecasting objective, the investors are skeptical about the forecast accuracy and assign a non-zero probability that the analyst is biased. Thus, the market price differs from the released forecast. Firms observing low market demand benefit from this uncertainty because: (i) unbiased forecasts are given a low weight in the price formation and (ii) biased forecasts are likely more favorable than the true state. Thus firms observing relatively low θ prefer to remain silent. And vice versa for firms observing high θ—they do not benefit from the investors' uncertainty and prefer to respond.

Proposition 2. *The late disclosure threshold $\theta^{\mathcal{L}}$ is decreasing in the probability of information endowment, p. There exists a cutoff $f^{\mathcal{L}} \in (0, \mu)$, such that, if $f < f^{\mathcal{L}}$, the late threshold $\theta^{\mathcal{L}}$ is decreasing in the probability q that the analyst is unbiased. If $f > f^{\mathcal{L}}$, the late disclosure threshold $\theta^{\mathcal{L}}$ is increasing in q.*

[35] Technically, the threshold depends on f. We suppress it to reduce clutter.

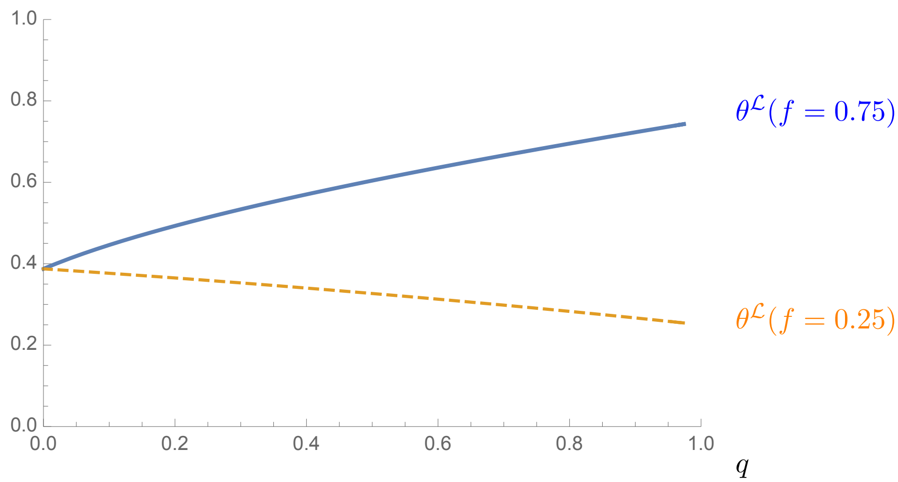

Figure 3: The late disclosure threshold $\theta^{\mathcal{L}}$ as a function of q
Numerical example with uniform distributions of θ and β, and $p = 0.6$.

The comparative statics of the late disclosure threshold with respect to the probability that the firm is endowed with information is straightforward: the higher p is, the harder it is for an informed firm to pretend to be uninformed. As a result, the firm discloses more often. The comparative statics with respect to q, depend on the forecast. When it is more likely that the analyst is unbiased, the investors put a greater weight on the forecast. Thus, the higher the forecast, the higher the nondisclosure price. As a result, the late disclosure threshold goes up. The opposite is true when the forecast is low. Then the nondisclosure price is lower which leads to a threshold decrease. Our result is formally stated in Proposition 2 and graphically illustrated in Figure 3.

A natural question that arises is whether the coverage by analysts with uncertain forecasting objective suppresses firms' disclosure.

Corollary 3. *There exists a cutoff $f^o \in (0, \theta^{\mathcal{D}})$ such that $\theta^{\mathcal{L}} < \theta^{\mathcal{D}}$ if $f < f^o$ and $\theta^{\mathcal{L}} > \theta^{\mathcal{D}}$ if $f > f^o$*

Our result is graphically illustrated in Figure 4. The firm responds to unfavorable forecasts ($f < f^o$) by disclosing values that would have been withheld in the absence

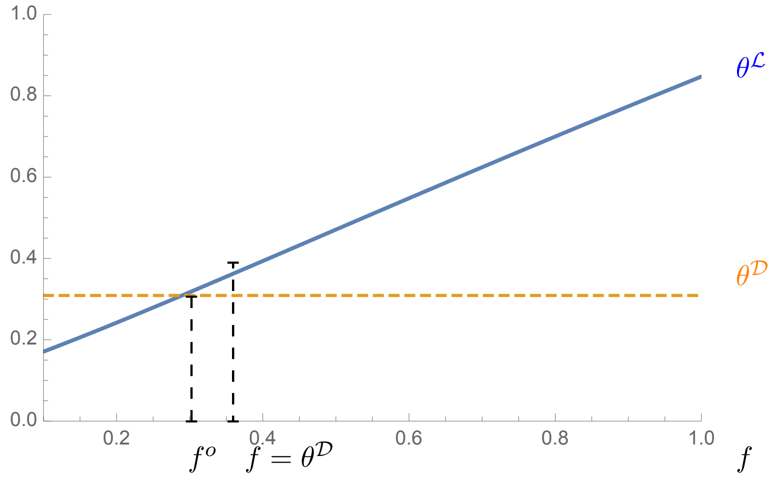

Figure 4: Comparison of $\theta^{\mathcal{D}}$ and $\theta^{\mathcal{L}}$
Numerical example with uniform distributions of θ and β, and $p = 0.6$ and $q = 0.4$.
Here, $f^o = 0.29$ and $\theta^{\mathcal{D}} = 0.31$.

of analyst coverage. Furthermore, following favorable forecasts ($f > f^o$), the firm withholds values that would have been disclosed in the absence of analyst coverage. Put differently, as one would expect, unfavorable forecasts encourage corporate disclosure, whereas favorable ones discourage disclosure. Interestingly, we find that, even when the analyst forecasts a value that equals precisely the benchmark threshold ($f = \theta^{\mathcal{D}}$), the disclosure threshold with analyst coverage still exceeds the benchmark one. We interpret this last observation as a manifestation of crowding-out effect of analyst coverage with uncertain forecasting objective.

An interesting question to ask is which forecasts induce the firm's response. It seems intuitive that firms respond only to negatively biased forecasts. Our next result shows that this intuition is not always correct.

Corollary 4. *There exists a cutoff* $\overline{f} \in (0, \mu)$*, such that* $\theta^{\mathcal{L}} > f$ *if* $f < \overline{f}$ *and* $\theta^{\mathcal{L}} < f$ *if* $f > \overline{f}$.

If the forecast is sufficiently negatively biased ($f < \overline{f}$), then the disclosure threshold exceeds it ($\theta^{\mathcal{L}} > f$); i.e., the firm withholds values that are *more favorable* than the

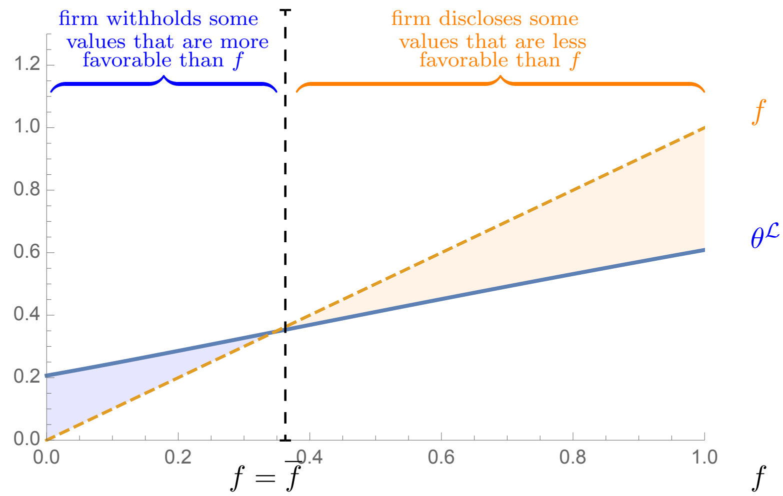

Figure 5: Late disclosure of unfavorable news and withholding of favorable news
Numerical example with uniform distributions of θ and β, $p = 0.6$ and $q = 0.4$. Here, $\overline{f} = 0.35$.

ones forecasted by the analyst. And vice versa: if the forecast is sufficiently positively biased ($f > \overline{f}$), the disclosure threshold falls short of it ($\theta^{\mathcal{L}} < f$); i.e., the firm discloses values that are *less favorable* than the ones forecasted by the analyst. This result is graphically illustrated in Figure 7.

Our result may seem perplexing: why would anyone withhold favorable news but reveal unfavorable news? The answer to this question lies in the way the investors react to statements made by analysts with uncertain forecasting objective. To fix ideas, suppose that the analyst is unbiased and forecasts exactly what the firm observes, i.e., $f = \theta$. Even though the analyst in this case is unbiased, the investors are uncertain about her type and the firm's information endowment. As a result, the price is a weighted average of the forecast, the prior μ, and the expected state, given that it is below the threshold. Hence the price has a sensitivity to the forecast that is strictly lower than 1. More importantly, because of the mean reversion, low forecasts lead to prices that exceed them, whereas high forecasts lead to prices that fall short of them. The result of Corollary 4 arises because, in equilibrium, the late disclosure threshold is equal to the price following nondisclosure. Note that the switching point is strictly below the mean μ because a nonzero weight is given to the expected project value, conditional on it being below the news, which is strictly lower than the prior.

4.2 Early disclosure

We continue with the analysis of the firm's voluntary disclosure under the early disclosure scenario. Because the firm's disclosure at date 2 precedes the analyst's forecast at date 3, the firm must decide whether to disclose or not without observing f. Thus the firm needs to form an expectation of the nondisclosure price, given the state it observed. Formally, the firm's expectation of the market price upon nondisclosure is $\mathbb{E}_f[P(\emptyset, f|\widehat{\theta})]|\theta]$, where $\mathbb{E}_f[\cdot|\theta]$ is the expectation taken by the firm over f, given the observed θ and a threshold $\widehat{\theta}$:

$$
\begin{aligned}
\mathbb{E}_f[P(f,\emptyset|\widehat{\theta})]|\theta] &= \Pr(i=U|\theta)\times\mathbb{E}_f[\mathbb{E}[\theta|\emptyset,\widehat{\theta},f]|i=U,\theta] \\
&\quad +\Pr(i=B|\theta)\times\mathbb{E}_f[\mathbb{E}[\theta|\emptyset,\widehat{\theta},f]|i=B,\theta].
\end{aligned}
$$

Because the forecast is not released yet, the observation of θ by the firm carries no information about the analyst's type. Thus $\Pr(i=U|\theta)=\Pr(i=U)=q$. Furthermore, the firm expects that forecasts made by unbiased analysts will reflect θ, i.e., $\mathbb{E}_f[\mathbb{E}[\theta|\emptyset,\widehat{\theta},f]|i=U,\theta]=\mathbb{E}_f[\mathbb{E}[\theta|\emptyset,\widehat{\theta},f]|f=\theta,\theta]$. Hence, with probability q the expected nondisclosure price is $P(\emptyset,f=\theta|\widehat{\theta})$. Similarly, with probability $\Pr(i=B|\theta)=\Pr(i=B)=1-q$, the firm expects that the analyst is biased and her forecast will equal β, i.e., $\mathbb{E}_f[\mathbb{E}[\theta|\emptyset,\widehat{\theta},f]|i=B,\theta]=\mathbb{E}_f[\mathbb{E}[\theta|\emptyset,\widehat{\theta},f]|f=\beta,\theta]$. It turns out that the expected nondisclosure price in this case is additively separable in the price that would prevail in the absence of analyst coverage, $P(\emptyset|\widehat{\theta})$, and an additional term that reflects the difference in the distributions of the state θ and the value β towards which analysts type $i=B$ are biased..

Lemma 4. *The expected nondisclosure price is given by* $\mathbb{E}_f[P(\emptyset, f|\widehat{\theta})]|\theta] = q \times P(\emptyset, f = \theta|\widehat{\theta}) + (1-q) \times \Big(P(\emptyset|\widehat{\theta}) + R(\Delta, \underline{\Delta}(\widehat{\theta}))\Big)$, *where* $R(\cdot)$ *is an increasing function of* $\Delta \equiv \mathbb{E}[\beta] - \mathbb{E}[\theta]$ *and* $\underline{\Delta}(\widehat{\theta}) \equiv \mathbb{E}[\beta|\beta < \widehat{\theta}] - \mathbb{E}[\theta|\theta < \widehat{\theta}]$.

If analysts type $i = B$ are more likely to be biased towards forecasting high values (as compared to the distribution of the true state) the nondisclosure price is affected positively. And vice versa, if analysts are more likely to be biased towards low values, the nondisclosure price is affected negatively. Our next corollary follows immediately.

Corollary 5. *First-order stochastically dominance of* $h(\cdot)$ *over* $g(\cdot)$ *has a positive effect on the expected nondisclosure price.*

In the special case where β is distributed exactly as θ, the informed firm, when deciding whether to disclose, considers only the effect of forecasts made by unbiased analysts.

Corollary 6. *Suppose that the uncertain bias* β *is distributed according to the same distribution as the state* θ*, i.e.,* $h(\cdot) = g(\cdot)$*. Then, when deciding whether to disclose an observed value, the firm disregards the effect that forecasts made by biased analysts have on the expected nondisclosure price.*

The reason behind our result is that, when $h(\cdot) = g(\cdot)$, the expectations of β and θ conditional on being below any value are identical. Thus $\Delta = 0$ and $\underline{\Delta} = 0$ so that $\mathbb{E}_f[\mathbb{E}[\theta|\emptyset, \widehat{\theta}, f]|f \neq \theta, \theta] = \mathbb{E}[\theta|\emptyset, \widehat{\theta}] = P(\emptyset|\widehat{\theta})$, which is independent of f. Put differently, in this special case, forecasts released by biased analysts act as noise and "wash out" in expectation. Hence only forecasts made by unbiased analysts affect the firm's disclosure decision.

We are now ready to formally establish the existence of a unique threshold equilibrium under the early disclosure scenario (superscript "$\mathcal{E}$").

Proposition 3. *There exists a unique threshold $\theta^{\mathcal{E}} \in (0,1)$, such that an informed firm discloses the observed state if $\theta \geq \theta^{\mathcal{E}}$ and withholds it otherwise.*

When deciding whether to disclose, an informed firm compares the price in case of disclosure with its expectation of the price in case of nondisclosure. As before, the price following disclosure, $P(\theta, f) = \theta$, is increasing in the observed state. However, in contrast to the benchmark case and the late disclosure scenario, for given investors' beliefs about the threshold strategy $\widehat{\theta} = \theta^{\mathcal{E}}$, the firm's expectation of the price in case of nondisclosure, $\mathbb{E}_f[P(\emptyset, f|\theta^{\mathcal{E}}]|\theta]$, is not constant—it is also increasing in θ. Nevertheless, as our result in Proposition 3 formally establishes, a threshold equilibrium continues to exist in the early disclosure scenario.

First, while the disclosure price increases at a rate of $\frac{\partial}{\partial\theta}P(\theta, f) = 1$, the expected nondisclosure price (for a given threshold) increases at a rate of $\frac{\partial}{\partial\theta}\mathbb{E}_f[P(\emptyset, f|\theta^{\mathcal{E}}]|\theta] < 1$. The reason is that, for a given conjectured disclosure threshold, the expected price following nondisclosure depends on the state only through the forecast whenever it is released by an unbiased analyst. Even when the analyst is unbiased, the investors are skeptical and assign a positive probability that the analyst is biased (as long as $q < 1$). As a result, as shown in (10), the price following nondisclosure is a weighted average of the forecast, the prior mean, and the expected state, given it is below the conjectured threshold. Hence the sensitivity to the forecast is strictly below 1. Second, and perhaps more important, the expected nondisclosure price when $f = 0$ is strictly above 0, whereas that following $f = 1$ is strictly below 1. These observations ensure the existence of a unique threshold equilibrium.

Proposition 4. *Suppose that the uncertain bias β is distributed according to the same distribution as the state θ, i.e., $h(\cdot) = g(\cdot)$. Then, the threshold $\theta^{\mathcal{E}}$ is decreasing in the probability that the firm is endowed with information, p, and increasing in the probability that the analyst is unbiased, q.*

Not surprisingly, the disclosure threshold $\theta^{\mathcal{E}}$ is decreasing in the probability p that the firm is endowed with information. As before, the higher p is, the higher are the beliefs of the investors that the firm is informed but chose to withhold its information. Hence the lower the weight the investors place on the prior expectation μ and the higher the weight they place on the expected state, conditional on being below the threshold $\theta^{\mathcal{E}}$ (this expectation is strictly lower than the prior μ). As a result, the expected nondisclosure price is lower, which drives the equilibrium disclosure threshold down and the firm discloses more often.

The comparative statics of the threshold $\theta^{\mathcal{E}}$ with respect to q is driven by the fact that the more likely it is that the analyst is unbiased, the more weight is shifted to the forecast f away from the prior expectation μ and the posterior expectation given that the state is below the threshold $\theta^{\mathcal{E}}$. By Corollary 6, when the state θ and the bias β are distributed according to the same distribution, $h(\cdot) = g(\cdot)$, the firm only considers unbiased forecasts. When selecting the equilibrium threshold, the firm focuses on the forecast that equals the threshold $\theta^{\mathcal{E}}$, which, for $h(\cdot) = g(\cdot)$, is below the prior μ. Thus shifting weight away from μ reduces the nondisclosure price and thereby the equilibrium disclosure threshold. However, this is more than outweighed by the opposite effect occurring because weight is shifted away from the posterior $\mathbb{E}[\theta|\theta < \theta^{\mathcal{E}}]$.

A natural next step is to consider how (potentially biased) forecasts affect the likelihood of voluntary disclosure. Given that, for a given disclosure threshold $\widehat{\theta}$, the probability of disclosure is $p(1 - G(\widehat{\theta}))$, it is enough to compare $\theta^{\mathcal{E}}$ with the benchmark $\theta^{\mathcal{D}}$.

Proposition 5. *Suppose that the uncertain bias β is distributed according to the same distribution as the state θ, i.e., $h(\cdot) = g(\cdot)$. Then, analyst coverage suppresses voluntary disclosures, i.e., $\theta^{\mathcal{E}} - \theta^{\mathcal{D}} \geq 0$. The more likely that the analyst is unbiased, the stronger the suppression effect, i.e., $\frac{\partial}{\partial q}(\theta^{\mathcal{E}} - \theta^{\mathcal{D}}) > 0$.*

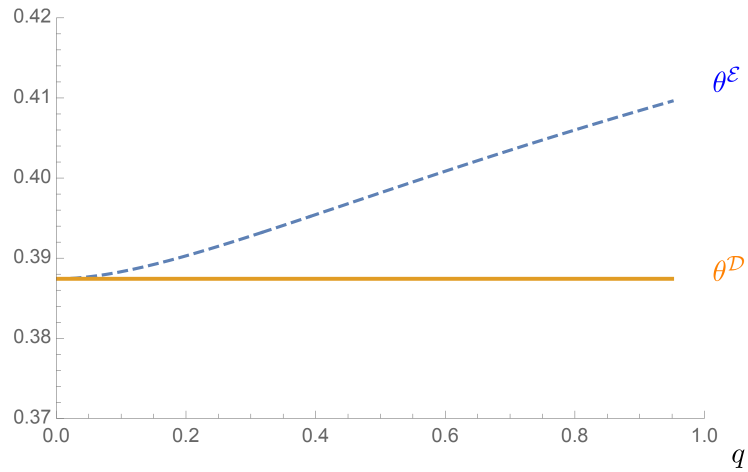

Figure 6: Illustration of the crowding-out effect of analyst coverage

Numerical example with uniform distributions of θ and β, and $p = 0.6$. Note that in the benchmark case there is no analyst coverage and therefore the disclosure threshold $\theta^{\mathcal{D}}$ is independent of q.

Our result is graphically illustrated in Figure 4. All types that withhold under the benchmark continue to withhold in the presence of analyst coverage. However, when facing analyst coverage, firms observing states $\theta \in [\theta^{\mathcal{D}}, \theta^{\mathcal{E}}]$ also withhold their information. Thus we predict that analyst coverage crowds out voluntary disclosure.

Our result implies that some firms are better off when their information is forecasted by a potentially biased analyst, rather than when they directly disclose it. The impetus for this result is the investors' uncertainty about the analyst forecasting objective and the firm's information endowment. Specifically, if the investors believe the forecast is unbiased, they react the same way as they would have reacted if the firm were to disclose the information. However, because of the reinforcement effect, if they believe that the forecast is biased, the investors assign a higher likelihood that the firm is uninformed and hence put greater weight on the prior μ.[36] Hence some firms observing values below the prior μ benefit from relying on the analyst to forecast those values.

[36]If the forecast is biased, the firm is more likely to be uninformed.

5 Strategic choice of timing

Until now we considered the timing of the corporate disclosures and the analysts forecast as exogenously given. We now discuss the strategic consideration of the players related to the timing of their actions.

5.1 The optimal timing of firm's disclosure

In this subsection, we consider the optimal timing of disclosure from the firm's perspective. If, at date 1, the firm observes $\theta > \lim_{f\to 1}\theta^{\mathcal{L}}$, then the firm is indifferent between making its disclosure decision early or late.[37] This is because, in either case, the firm ends up disclosing and the price equals the observed state. However, for any observed value $\theta \in [\theta^{\mathcal{E}}, \lim_{f\to 1}\theta^{\mathcal{L}}]$ the firm prefers to delay the disclosure decision. This is because, by delaying, the firm may end up withholding the observed value (if, at date 4, this value is below $\theta^{\mathcal{L}}$). The equilibrium market price following nondisclosure is higher than the observed value. Thus the firm can only benefit from the delay. Our discussion is summarized below.

Corollary 7. *Suppose $h(\cdot) = g(\cdot)$. Firms observing favorable information can only benefit from delaying their disclosure decision.*

How about firms observing unfavorable market demand information? Clearly, a firm observing $\theta < \theta^{\mathcal{E}}$ has nothing to disclose at date 2. When the firm cannot credibly commit that it has nothing to disclose at date 2, it has no choice but remain silent. By doing so, it may end up benefiting or losing. Specifically, if $\theta^{\mathcal{L}} < \theta < \theta^{\mathcal{E}}$, the firm will have to disclose at date 4 and end up with a market price that is strictly lower than the one achievable in the early disclosure regime. On the other hand, if $\theta^{\mathcal{L}} > \theta^{\mathcal{E}} > \theta$, the

[37]Note that since $\theta^{\mathcal{E}}$ is set in expectation over f (conditional on θ), it has to be the case that $\theta^{\mathcal{E}} \leq \lim_{f\to 1}\theta^{\mathcal{L}}$.

firm will remain silent also at date 4 and will be lumped together with higher types, thereby obtaining a higher market price. Given that in expectation $\mathbb{E}[\theta^{\mathcal{E}}] = \theta^{\mathcal{L}}$, the firm observing unfavorable information should be indifferent. Future research will explore how the ability to commit to nondisclosure at date 2 changes the strategic choice of firms observing unfavorable information. Of specific interest is how early commitment to nondisclosure affects the investors' beliefs about the firm's information endowment.

5.2 The optimal timing of forecast release by a biased analyst

In this subsection, we discuss a variation of our model where the corporate disclosure timing is fixed but the analyst can choose the timing of her forecast. An unbiased analyst aiming to issue as precise as possible forecast is indifferent about when the forecast is released. However, this may not be the case for a biased analyst. Until now we were a bit casual about the source of analyst's bias and only pointed out in footnote 28 that the biased analyst's objective could be to minimize the difference between the market price and a specific unknown value. Such objective could be represented by:

$$\pi_B = -(P(d, f) - m(\beta))^2,$$

where $m(\beta)$ represents the market price level towards which the analyst is biased and, as before, β is the analyst's unknown bias. As already discussed, the market price in case of disclosure is $P(\theta, f) = \theta$ so that the biased analyst has no way of maximizing her payoff. Thus the objective of a biased analyst boils down to (i) preempting the firm's disclosure and (ii) issuing a forecast that satisfies $P(\emptyset, f) = m(\beta)$. Because the biased analyst knows her (biased) forecast and thus the late disclosure threshold, the choice of forecast timing to preempt corporate disclosure can be simplified to comparison of the late disclosure threshold $\theta^{\mathcal{L}}$, with the early disclosure threshold $\theta^{\mathcal{E}}$.

Corollary 8. *Suppose $h(\cdot) = g(\cdot)$. There exists a cutoff $\beta^{delay} \in (0, \mu)$, such that only analysts with bias $\beta < \beta^{delay}$ delay their forecasts.*

While $\theta^{\mathcal{E}}$ is set in expectation over f (and is thereby constant), $\theta^{\mathcal{L}}$ is set for a given forecast and is increasing in f. If the biased analyst expects to forecast $f = \beta$ below the cutoff β^{delay}, the threshold under the early disclosure scenario, exceeds the one under the late disclosure scenario. Then, all else equal, delaying the forecast maximizes the probability that the firm will remain silent and the analyst will be able to maximize her payoff.

6 Empirical predictions

In the remainder of the paper, we discuss several implications of our late disclosure scenario that can help explain the empirical evidence about the association between stock price declines and corporate disclosures of (relatively) favorable information as well as the association between stock price increases and firm's disclosure of (relatively) unfavorable news. We slightly deviate from our assumption that the investors price the firm only at date 5 and assume instead that they set a price after the arrival of every piece of information, i.e., at dates 3 and 2 (with early disclosure) or 4 (with late disclosure).[38]

To fix ideas, let P_t denote the price at date t. Initially, the price before any information arrives is $P_1 = \mathbb{E}[\theta] = \mu$. Immediately after observing the forecast and before any response (or lack thereof) by the firm, the price is $P_3(f) = \mathbb{E}[\theta|f] = qf + (1-q)\mu$. Note that, depending on whether the forecast is above or below the prior mean, the

[38]This change in assumption does not affect our results qualitatively but allows us to track the market price after the arrival of every piece of information, as is typical in stock markets.

price at date 3 may be above or below the initial price, i.e.,

$$P_1 < P_3(f) \Leftrightarrow f > \mu.$$

Hence, as one would expect, favorable news increase the price, and unfavorable news—decrease it.

At date 4, the price depends on whether the firm responds. If it does, the price is $P_4(f, v) = \theta$; otherwise, the price is $P_4(f, \emptyset) = P(\emptyset, f|\theta^{\mathcal{L}})$, as defined in (10).

Corollary 9. *The price following no-response to the forecast always decreases, i.e., $P_4(f, \emptyset) < P_3(f)$ for any f. The price following a response may decrease or increase, depending on the value reported in the news, i.e., $P_4(f, v) < P_3(f) \Leftrightarrow f > \frac{v-(1-q)\mu}{q}$.*

Our result implies that, as one would expect, when the firm does not respond to the forecast, the price decreases. This is because, when the firm does not respond, the investors conclude that there is a chance the firm observed an even lower value. However, the price may also decrease when the firm responds. The impetus for this result is that, after the forecast, the firm anticipates a further decrease in the price if it does not respond. To avoid this, it responds by disclosing values that, while lower than the price from date 3, exceed the price that would have prevailed if it had remained silent.

Our results in this subsection may explain the empirical evidence consistent with price increases following unfavorable information arrival and price decreases following favorable information. Relatedly, Sletten (2011) shows that, in response to unfavorable external news, firms disclose information. While in most cases, following the corporate disclosure, stock price increases, in some cases the price decreases.

7 Concluding Remarks

We study the interaction between corporate disclosures and forecasts issued by analysts with uncertain forecasting objective. We find that analyst coverage may suppress voluntary disclosure when the latter happens before or simultaneously with the release of the news. Otherwise, when the disclosure occurs in response to the forecast, the firm may reveal information that is less favorable than the analyst's forecast but withhold more favorable information. Our results shed light on the empirical evidence about stock price decreases following favorable exogenous news and stock price increases following unfavorable news. Future work may consider background communication between firms and analysts.

Appendix C - Proofs

Proof of Lemma 0: Follows directly from the proof in Dye (1985) and Jung and Kwon (1998) and is omitted. For future reference, the threshold $\theta^{\mathcal{D}}$ satisfies $\theta^{\mathcal{D}} = \Pr(\text{NoInf}|d = \emptyset, \theta^{\mathcal{D}}) \cdot \mu + (1 - \Pr(\text{NoInf}|d = \emptyset, \theta^{\mathcal{D}})) \cdot \mathbb{E}[\theta|\theta \leq \theta^{\mathcal{D}}]$. □

Proof of Lemma 1: The proof follows directly from the discussion in the text and is omitted. □

Proof of Lemma 2: The proof follows from the arguments leading to Lemma 1. □

Proof of Corollary 2: The proof follows from the fact that for any $\widehat{\theta} < 1$ we get $q < \delta(\widehat{\theta}) = \frac{q}{q+(1-q)H(\widehat{\theta})}$. Furthermore, if $\widehat{\theta} = 1$ we get $q = \delta(\widehat{\theta}) = \frac{q}{q+(1-q)H(\widehat{\theta})}$. Lastly, $0 < \delta(\widehat{\theta}) = \frac{q}{q+(1-q)H(\widehat{\theta})}$. (Recall that $q > 0$, by assumption.) Note that all expressions above are independent of β. □

Proof of Lemma 3: First, we note that $\Pr(\mathit{NoInf}|\emptyset, \widehat{\theta}, f) = 1 - \Pr(\mathit{Inf} \cap \theta < \widehat{\theta}|\emptyset, \widehat{\theta}, f)$. Furthermore,

$$\Pr(\mathit{Inf} \cap \theta < \widehat{\theta}|\emptyset, \widehat{\theta}, f < \widehat{\theta}) = \frac{pG(\widehat{\theta})}{1 - p + pG(\widehat{\theta})} + \frac{qp(1-p)(1 - H(\widehat{\theta}))}{1 - p + pG(\widehat{\theta})} > \Pr(\mathit{Inf} \cap \theta < \widehat{\theta}|\emptyset, \widehat{\theta}).$$

Furthermore,

$$\Pr(\mathit{Inf} \cap \theta < \widehat{\theta}|\emptyset, \widehat{\theta}, f > \widehat{\theta}) \quad = \quad (1 - q)\frac{pG(\widehat{\theta})}{1 - p + pG(\widehat{\theta})} < \Pr(\mathit{Inf} \cap \theta < \widehat{\theta}|\emptyset, \widehat{\theta}).$$

Combining our observations yields the result. □

Proof of Proposition 1: Since f is fixed, for any (assumed) threshold $\theta^{\mathcal{L}}$ the price upon nondisclosure, is constant while the price upon disclosure increases in the disclosed value, thus a threshold equilibrium may exist. The equilibrium condition is

$$\begin{aligned} N^{\mathcal{L}}(\theta^{\mathcal{L}}, f) &= P(\emptyset, f|\theta^{\mathcal{L}}) - P(\theta = \theta^{\mathcal{L}}, f) \\ &= \gamma(\theta^{\mathcal{L}})\mu + (1-\gamma(\theta^{\mathcal{L}}))\mathbb{E}[\theta|\theta \le \theta^{\mathcal{L}}] + \gamma(\theta^{\mathcal{L}})q(f-\mu) \\ &\quad +(1-\gamma(\theta^{\mathcal{L}}))\delta(\theta^{\mathcal{L}})(f - \mathbb{E}[\theta|\theta \le \theta^{\mathcal{L}}]) - \theta^{\mathcal{L}} \\ &= 0 \end{aligned}$$

We note that $N^{\mathcal{L}}(\cdot)$ is decreasing in $\theta^{\mathcal{L}}$ and that $\lim_{\theta^{\mathcal{L}}\to 0} N^{\mathcal{L}}(\cdot) = \mu + q(f-\mu) > 0$, whereas $\lim_{\theta^{\mathcal{L}}\to 1} N^{\mathcal{L}}(\cdot) = (1-p)\mu + p\mu + (1-p)q(f-\mu) + pq(f-\mu) - 1 < 0$. Therefore a unique threshold equilibrium exists, whereby the conjectured disclosure threshold $\theta^{\mathcal{L}} \in (0,1)$ is correct. $\square$

Proof of Proposition 2: Using the proof of Proposition 1 and the Implicit Function Theorem,

$$\frac{d}{dp}\theta^{\mathcal{L}} \;\propto\; \frac{\partial}{\partial p}N^{\mathcal{L}}(\cdot) \propto \left(1-\delta(\theta^{\mathcal{L}})\right)\mathbb{E}[\theta|\theta \le \theta^{\mathcal{L}}] - (1-q)\mu < 0,$$

because $\mathbb{E}[\theta|\theta \le \theta^{\mathcal{L}}] < \mu$ and $1-\delta(\theta^{\mathcal{L}}) < 1-q$. Furthermore,

$$\frac{d}{dq}\theta^{\mathcal{L}} \;\propto\; \frac{\partial}{\partial q}N^{\mathcal{L}}(\cdot) = \gamma(\theta^{\mathcal{L}})(f-\mu) + (1-\gamma(\theta^{\mathcal{L}}))\frac{\partial}{\partial q}\delta(\theta^{\mathcal{L}})(f - \mathbb{E}[\theta|\theta \le \theta^{\mathcal{D}}]) \equiv n^{\mathcal{L}}(f).$$

We note that $n^{\mathcal{L}}(f)$ is increasing in f, $\lim_{f\to 0} n^{\mathcal{L}}(f) < 0$ and $\lim_{f\to\mu} n^{\mathcal{L}}(f) > 0$. Hence, there exists a cutoff $f^{\mathcal{L}} \in (0,\mu)$ such that $\theta^{\mathcal{L}}$ is increasing in q if $f > f^{\mathcal{L}}$ and decreasing in q if $f < f^{\mathcal{L}}$. $\square$

Proof of Corollary 3: By the proof of Proposition 1, the threshold $\theta^{\mathcal{L}}$ satisfies

$N^{\mathcal{L}}(\theta^{\mathcal{L}}, f) = 0$. By the proof of Lemma 0, the threshold $\theta^{\mathcal{D}}$ satisfies

$$\gamma(\theta^{\mathcal{D}})\mu + (1 - \gamma(\theta^{\mathcal{D}})\mathbb{E}[\theta|\theta \leq \theta^{\mathcal{D}}] - \theta^{\mathcal{D}} = 0. \tag{11}$$

To show that $\theta^{\mathcal{D}} > \theta^{\mathcal{L}}$ ($\theta^{\mathcal{D}} < \theta^{\mathcal{L}}$) it is enough to show that $N^{\mathcal{L}}(\theta^{\mathcal{D}}, f) < 0$ ($N^{\mathcal{L}}(\theta^{\mathcal{D}}, f) > 0$). Using (11), we get

$$N^{\mathcal{L}}(\theta^{\mathcal{D}}, f) \;=\; \gamma(\theta^{\mathcal{D}})q(f - \mu) + (1 - \gamma(\theta^{\mathcal{D}}))\delta(\theta^{\mathcal{D}})(f - \mathbb{E}[\theta|\theta \leq \theta^{\mathcal{D}}])$$

We note that $N^{\mathcal{L}}(\cdot)$ is monotonically increasing in f. Furthermore, $\lim_{f\to 0} N^{\mathcal{L}}(\cdot) < 0$, whereas

$$\begin{aligned}
\lim_{f\to\theta^{\mathcal{D}}} N^{\mathcal{L}}(\cdot) &= \gamma(\theta^{\mathcal{D}})q(\theta^{\mathcal{D}} - \mu) + (1 - \gamma(\theta^{\mathcal{D}}))\delta(\theta^{\mathcal{D}})(\theta^{\mathcal{D}} - \mathbb{E}[\theta|\theta \leq \theta^{\mathcal{D}}]) \\
&= \gamma(\theta^{\mathcal{D}})q\theta^{\mathcal{D}} - \gamma(\theta^{\mathcal{D}})q\mu + (1 - \gamma(\theta^{\mathcal{D}}))\delta(\theta^{\mathcal{D}})\theta^{\mathcal{D}} \\
&\quad -(1 - \gamma(\theta^{\mathcal{D}}))\delta(\theta^{\mathcal{D}})\mathbb{E}[\theta|\theta \leq \theta^{\mathcal{D}}] \\
&\quad -(1 - \gamma(\theta^{\mathcal{D}}))q\mathbb{E}[\theta|\theta \leq \theta^{\mathcal{D}}] + (1 - \gamma(\theta^{\mathcal{D}}))q\mathbb{E}[\theta|\theta \leq \theta^{\mathcal{D}}] \\
&= \theta^{\mathcal{D}}(\gamma(\cdot)q + (1 - \gamma(\cdot))\delta(\cdot)) - q\underbrace{(\gamma(\cdot)\mu + (1 - \gamma(\cdot))\mathbb{E}[\theta|\theta \leq \theta^{\mathcal{D}}])}_{=\theta^{\mathcal{D}}} \\
&\quad -(1 - \gamma(\cdot))\mathbb{E}[\theta|\theta \leq \theta^{\mathcal{D}}](\delta(\cdot) - q) \\
&= \theta^{\mathcal{D}}(1 - \gamma(\cdot))(\delta(\cdot) - q) - (1 - \gamma(\cdot))(\delta(\cdot) - q)\mathbb{E}[\theta|\theta \leq \theta^{\mathcal{D}}] \\
&\propto \theta^{\mathcal{D}} - \mathbb{E}[\theta|\theta \leq \theta^{\mathcal{D}}] \\
&> 0.
\end{aligned}$$

Thus we conclude that there exists $f^o \in (0, \theta^{\mathcal{D}})$, such that $\theta^{\mathcal{D}} > \theta^{\mathcal{L}}$ when $f < f^o$ and $\theta^{\mathcal{D}} < \theta^{\mathcal{L}}$ when $f > f^o$. $\square$

Proof of Corollary 4: From the proof of Proposition 1 we know that $\theta^{\mathcal{L}}$ satisfies:

$N^{\mathcal{L}}(\theta^{\mathcal{L}}, f) = 0$. Using the Implicit Function Theorem, $\frac{d\theta^{\mathcal{L}}}{df} = -\frac{\frac{\partial N^{\mathcal{L}}(\cdot)}{\partial f}}{\frac{\partial N^{\mathcal{L}}(\cdot)}{\partial \theta^{\mathcal{L}}}} \propto \frac{\partial N^{\mathcal{L}}(\cdot)}{\partial f} < 0$. Because for $f = 0$ we get that $\theta^{\mathcal{L}} > 0$ and for $f = \mu$ we get that $\theta^{\mathcal{L}} < \mu$, then there exist $\overline{f} \in (0, \mu)$ such that $\theta^{\mathcal{L}} = \overline{f}$ and for any $f < \overline{f}$ we get that $f < \theta^{\mathcal{L}}$ and vice versa for $\overline{f} < f$. □

Proof of Lemma 4: Using (10) and Lemma 1, the price following nondisclosure can be expressed as:

$$P(\emptyset, f|\widehat{\theta}) = (1-q)\gamma(\cdot) \times \mu + (1-\gamma(\cdot))(1-\tau) \times \mathbb{E}[\theta|\theta < \widehat{\theta}] + \Big(\gamma(\cdot)q + (1-\gamma(\cdot))\tau\Big) \times f,$$

where $\tau = \mathbb{1}_{f<\overline{\theta}}\delta(\cdot)$. The expected nondisclosure price is equal to

$$\mathbb{E}_f[P(\emptyset, f|\widehat{\theta})|\theta] \;\; = \;\; \Pr(i = U)P(\emptyset, f = \theta|\widehat{\theta}) + \Pr(i = B)\mathbb{E}_f[P(\emptyset, f|\widehat{\theta})|\theta, f = \beta].$$

Let us focus on $\mathbb{E}_f[P(\emptyset, f|\widehat{\theta})|\theta, f = \beta]$. When taking expectation we have to consider two cases:

Case (i): The biased forecast happens to be below the threshold $\Rightarrow f = \beta < \widehat{\theta}$;

Case (ii): The biased forecast happens to be above the threshold $\Rightarrow f = \beta > \widehat{\theta}$.

Thus we can express

$$\mathbb{E}_f[P(\emptyset, f|\widehat{\theta})|f \neq \theta, \theta] = \Pr(f < \widehat{\theta}|f \neq \theta) \times \underline{\phi} + \Pr(f > \widehat{\theta}|f \neq \theta) \times \overline{\phi},$$

where

$$\begin{aligned}
\underline{\phi} \;\; &\equiv \;\; (1-q)\gamma(\cdot) \times \mu + (1-\gamma(\cdot))(1-\delta(\cdot)) \times \mathbb{E}[\theta|\theta < \widehat{\theta}] \\
&\quad + \Big(\gamma(\cdot)q + (1-\gamma(\cdot))\delta(\cdot)\Big) \times \mathbb{E}[f|f = \beta < \overline{\theta}], \\
\overline{\phi} \;\; &\equiv \;\; (1-q)\gamma(\cdot) \times \mu + (1-\gamma(\cdot)) \times \mathbb{E}[\theta|\theta < \widehat{\theta}] + \gamma(\cdot)q \times \mathbb{E}[f|f = \beta > \overline{\theta}].
\end{aligned}$$

Noting that $\Pr(f < \widehat{\theta} | f \neq \theta) = H(\widehat{\theta})$ and $\Pr(f > \widehat{\theta} | f \neq \theta) = 1 - H(\widehat{\theta})$ and rearranging,

$$\begin{aligned}
\mathbb{E}_f[P(\emptyset, f|\widehat{\theta})|f \neq \theta, \theta] \quad = \quad & (1-q)\gamma(\cdot) \times \mu \\
& + \gamma(\cdot) q \Big(\underbrace{H(\widehat{\theta})\mathbb{E}[\beta|\beta < \overline{\theta}] + (1 - H(\widehat{\theta}))\mathbb{E}[\beta|\beta > \overline{\theta}]}_{=\mathbb{E}[\beta]} \Big) \\
& + (1-\gamma(\cdot)) \Big(H(\widehat{\theta})(1 - \delta(\cdot)) + 1 - H(\widehat{\theta}) \Big) \times \mathbb{E}[\theta|\theta < \widehat{\theta}] \\
& + (1-\gamma(\cdot))\delta(\cdot) H(\widehat{\theta}) \mathbb{E}[\beta|\beta < \widehat{\theta}].
\end{aligned}$$

Simplifying and rearranging,

$$\mathbb{E}_f[P(\emptyset, f|\widehat{\theta})|f \neq \theta, \theta] \quad = \quad \underbrace{\gamma(\cdot)\mu + (1-\gamma(\cdot))\mathbb{E}[\theta|\theta < \widehat{\theta}]}_{=P(\emptyset|\widehat{\theta})} + R(\Delta, \underline{\Delta}(\widehat{\theta})),$$

where $R(\Delta, \underline{\Delta}(\widehat{\theta})) \equiv \gamma(\cdot) q \Delta + (1-\gamma(\cdot))\delta(\cdot) H(\widehat{\theta}) \underline{\Delta}(\widehat{\theta})$, $\Delta \equiv \mathbb{E}[\beta] - \mathbb{E}[\theta]$ and $\underline{\Delta}(\widehat{\theta}) \equiv \mathbb{E}[\beta|\beta < \widehat{\theta}] - \mathbb{E}[\theta|\theta < \widehat{\theta}]$. Combining our observations yields the result. □

Proof of Corollary 5: Follows immediately from the discussion in the main text. □

Proof of Corollary 6: Follows immediately from the discussion in the main text. □

Proof of Proposition 3: Using Lemma 4, we can express the equilibrium condition as:

$$N^{\mathcal{E}}(\theta^{\mathcal{E}}) \equiv q \times P(\emptyset, f = \theta|\widehat{\theta}) + (1-q) \times \Big(P(\emptyset|\widehat{\theta}) + R(\Delta, \underline{\Delta}(\widehat{\theta})) \Big) - \theta^{\mathcal{E}} = 0.$$

Rearranging,

$$
\begin{aligned}
&N^{\mathcal{E}}(\theta^{\mathcal{E}}) = \\
&\quad q\Big((1-q)\gamma(\cdot)\mu + (1-\gamma(\cdot))(1-\delta(\cdot))\mathbb{E}[\theta|\theta < \theta^{\mathcal{E}}] + \Big(\gamma(\cdot)q + (1-\gamma(\cdot))\delta(\cdot)\Big)\theta^{\mathcal{E}}\Big) \\
&\quad +(1-q)\Big(\gamma(\cdot)\mu + (1-\gamma(\cdot))\mathbb{E}[\theta|\theta < \theta^{\mathcal{E}}] + \gamma(\cdot)q\Delta + (1-\gamma(\cdot))\delta(\cdot)H(\theta^{\mathcal{E}})\underline{\Delta}(\theta^{\mathcal{E}})\Big) \\
&\quad -\theta^{\mathcal{E}}
\end{aligned}
$$

Simplifying,

$$
\begin{aligned}
N^{\mathcal{E}}(\theta^{\mathcal{E}}) &= \gamma(\cdot)\mu + (1-\gamma(\cdot))\mathbb{E}[\theta|\theta < \theta^{\mathcal{E}}] + q^2(\theta^{\mathcal{E}} - \mu)\gamma(\cdot) \\
&\quad +q\delta(\cdot)(1-\gamma(\cdot))(\theta^{\mathcal{E}} - \mathbb{E}[\theta|\theta < \theta^{\mathcal{E}}]) \\
&\quad +(1-q)q\gamma(\cdot)\Delta + (1-q)(1-\gamma(\cdot))\delta(\cdot)H(\theta^{\mathcal{E}})\underline{\Delta}(\theta^{\mathcal{E}})
\end{aligned}
$$

We observe that $N^{\mathcal{E}}(\theta^{\mathcal{E}})$ is decreasing in $\theta^{\mathcal{E}}$ and that

$$
\begin{aligned}
\lim_{\theta^{\mathcal{E}} \to 0} N^{\mathcal{E}}(\theta^{\mathcal{E}}) &= \mu - q^2\mu + (1-q)q\Delta = (1-q)(\mu + qE[\beta]) > 0 \\
\lim_{\theta^{\mathcal{E}} \to 1} N^{\mathcal{E}}(\theta^{\mathcal{E}}) &= (1-p)\mu + p\mu + q^2(1-\mu)(1-p) \\
&\quad +q^2p(1-\mu) + (1-q)q(1-p)\Delta + (1-q)pq\Delta - 1 \\
&= \mu + q^2(1-\mu) + q(1-q)\Delta - 1 \\
&= (1-q)\mu + q^2(1-\mathbb{E}[\beta]) - 1 \\
&< (1-q)\mu + q^2 - 1 < 0.
\end{aligned}
$$

Thus, there exists a unique threshold, $\theta^{\mathcal{E}} \in (0,1)$, such that $N^{\mathcal{E}}(\theta^{\mathcal{E}}) = 0$. Any type observing $\theta < \theta^{\mathcal{E}}$ is strictly better off withholding and any type observing $\theta > \theta^{\mathcal{E}}$ is strictly better off disclosing. $\square$

Proof of Proposition 4: Using the proof of Proposition 3, the disclosure threshold $\theta^{\mathcal{E}}$ is defined by $N^{\mathcal{E}}(\theta^{\mathcal{E}}) = 0$, where $N^{\mathcal{E}}(\cdot)$ is a decreasing function. Hence, using the Implicit Function Theorem, $\frac{d\theta^{\mathcal{E}}}{dp} = -\frac{\frac{\partial N^{\mathcal{E}}(\cdot)}{\partial p}}{\frac{\partial N^{\mathcal{E}}(\cdot)}{\partial \theta^{\mathcal{E}}}} \propto \frac{\partial N^{\mathcal{E}}(\cdot)}{\partial p}$ and $\frac{d\theta^{\mathcal{E}}}{dq} = -\frac{\frac{\partial N^{\mathcal{E}}(\cdot)}{\partial q}}{\frac{\partial N^{\mathcal{E}}(\cdot)}{\partial \theta^{\mathcal{E}}}} \propto \frac{\partial N^{\mathcal{E}}(\cdot)}{\partial q}$. We note that when $g(\cdot) = h(\cdot)$, we have $\theta^{\mathcal{E}} < \mu$. It is easy to verify that

$$\begin{aligned}
\frac{\partial N^{\mathcal{E}}(\cdot)}{\partial p} &= \frac{\partial}{\partial p}\left(\frac{1-p}{1-p+pG(\theta^{\mathcal{E}})}\right)(1-q^2)(\mu-\theta^{\mathcal{E}}) \\
&\quad + \frac{\partial}{\partial p}\left(\frac{pG(\widehat{\theta})}{1-p+pG(\theta^{\mathcal{E}})}\right)\left(1-\frac{q^2}{q+(1-q)G(\theta^{\mathcal{E}})}\right)\left(\mathbb{E}[\theta|\theta \le \theta^{\mathcal{E}}] - \theta^{\mathcal{E}}\right) \\
&= -\frac{G(\theta^{\mathcal{E}})}{(1-p+pG(\theta^{\mathcal{E}}))^2}(1-q^2)(\mu-\theta^{\mathcal{E}}) \\
&\quad + \frac{G(\theta^{\mathcal{E}})}{(1-p+pG(\theta^{\mathcal{E}}))^2}\left(1-\frac{q^2}{q+(1-q)G(\theta^{\mathcal{E}})}\right)\left(\mathbb{E}[\theta|\theta \le \theta^{\mathcal{E}}] - \theta^{\mathcal{E}}\right) \\
&\propto -(1-q^2)(\mu-\theta^{\mathcal{E}}) - \left(1-\frac{q^2}{q+(1-q)G(\theta^{\mathcal{E}})}\right)\left(\theta^{\mathcal{E}} - \mathbb{E}[\theta|\theta \le \theta^{\mathcal{E}}]\right) < 0.
\end{aligned}$$

Hence, $\frac{d\theta^{\mathcal{E}}}{dp} < 0$. Furthermore,

$$\begin{aligned}
\frac{\partial N^{\mathcal{E}}(\cdot)}{\partial q} &= \frac{\partial}{\partial q}(1-q^2)\left(\frac{1-p}{1-p+pG(\theta^{\mathcal{E}})}\right)(\mu-\theta^{\mathcal{E}}) \\
&\quad + \frac{\partial}{\partial q}\left(1-\frac{q^2}{q+(1-q)G(\theta^{\mathcal{E}})}\right)\left(\frac{pG(\theta^{\mathcal{E}})}{1-p+pG(\theta^{\mathcal{E}})}\right)\left(\mathbb{E}[\theta|\theta \le \theta^{\mathcal{E}}] - \theta^{\mathcal{E}}\right).
\end{aligned}$$

We can restate the equilibrium condition as:

$$\left(\frac{1-p}{1-p+pG(\theta^{\mathcal{E}})}\right)(\mu-\theta^{\mathcal{E}}) = \left(\frac{pG(\theta^{\mathcal{E}})}{1-p+pG(\theta^{\mathcal{E}})}\right)\left(\theta^{\mathcal{E}} - \mathbb{E}[\theta|\theta \le \theta^{\mathcal{E}}]\right)\left(\frac{1-\frac{q^2}{q+(1-q)G(\theta^{\mathcal{E}})}}{1-q^2}\right)$$

Substituting,

$$\begin{aligned}
\frac{\partial N^{\mathcal{E}}(\cdot)}{\partial q} &= \frac{\partial}{\partial q}(1-q^2)\left(\frac{pG(\theta^{\mathcal{E}})}{1-p+pG(\theta^{\mathcal{E}})}\right)\left(\theta^{\mathcal{E}}-\mathbb{E}[\theta|\theta\leq\theta^{\mathcal{E}}]\right)\left(\frac{1-\frac{q^2}{q+(1-q)G(\theta^{\mathcal{E}})}}{1-q^2}\right) \\
&\quad -\frac{\partial}{\partial q}\left(1-\frac{q^2}{q+(1-q)G(\theta^{\mathcal{E}})}\right)\left(\frac{pG(\theta^{\mathcal{E}})}{1-p+pG(\theta^{\mathcal{E}})}\right)\left(\theta^{\mathcal{E}}-\mathbb{E}[\theta|\theta\leq\theta^{\mathcal{E}}]\right) \\
&\propto -2q\left(\frac{1-\frac{q^2}{q+(1-q)G(\theta^{\mathcal{E}})}}{1-q^2}\right)+\frac{q(q-G(\theta^{\mathcal{E}})(q-2))}{(q+(1-q)G(\theta^{\mathcal{E}}))^2} \\
&\propto -2(q+(1-q)G(\theta^{\mathcal{E}}))^2+2q^2(q+(1-q)G(\theta^{\mathcal{E}})) \\
&\quad +(1-q^2)(q(1-G(\theta^{\mathcal{E}}))+2G(\theta^{\mathcal{E}})) \\
&= 2(q+(1-q)G(\theta^{\mathcal{E}}))(-(1-q)q-(1-q)G(\theta^{\mathcal{E}})) \\
&\quad +(1-q)(1+q)(q(1-G(\theta^{\mathcal{E}}))+2G(\theta^{\mathcal{E}})) \\
&\propto -2(q+(1-q)G(\theta^{\mathcal{E}}))(q+G(\theta^{\mathcal{E}}))+(1+q)(q(1-G(\theta^{\mathcal{E}}))+2G(\theta^{\mathcal{E}}))>0
\end{aligned}$$

because $0<q<1$ and $0<G(\theta^{\mathcal{E}})<1$. Hence, $\frac{d\theta^{\mathcal{E}}}{dq}>0$. □

Proof of Proposition 5: Taking into account that $\theta^{\mathcal{D}}=\frac{(1-p)\mu+pG(\theta^{\mathcal{D}})\mathbb{E}[\theta|\theta\leq\theta^{\mathcal{D}}]}{1-p+pG(\theta^{\mathcal{D}})}$, and using the proof of Proposition 3,

$$\begin{aligned}
N^{\mathcal{E}}(\theta^{\mathcal{D}}) &\propto (1-p)(1-q^2)(\mu-\theta^{\mathcal{D}}) \\
&\quad +pG(\theta^{\mathcal{D}})\left(1-\frac{q^2}{q+(1-q)G(\theta^{\mathcal{D}})}\right)\left(\mathbb{E}[\theta|\theta\leq\theta^{\mathcal{D}}]-\theta^{\mathcal{D}}\right) \\
&\propto \frac{(1-p)pG(\theta^{\mathcal{D}})}{1-p+pG(\theta^{\mathcal{D}})}(1-q^2)(\mu-\mathbb{E}[\theta|\theta\leq\theta^{\mathcal{D}}]) \\
&\quad -\frac{(1-p)pG(\theta^{\mathcal{D}})}{1-p+pG(\theta^{\mathcal{D}})}\left(1-\frac{q^2}{q+(1-q)G(\theta^{\mathcal{D}})}\right)\left(\mu-\mathbb{E}[\theta|\theta\leq\theta^{\mathcal{D}}]\right) \\
&\propto (1-q^2)-\left(1-\frac{q^2}{q+(1-q)G(\theta^{\mathcal{D}})}\right) \\
&\propto 1-(q+(1-q)G(\theta^{\mathcal{D}}))>0.
\end{aligned}$$

It follows that a firm observing $\theta^{\mathcal{D}}$ is better off withholding its information, i.e, $\theta^{\mathcal{E}}-\theta^{\mathcal{D}}>$

0. The comparative statics, $\frac{\partial}{\partial q}(\theta^{\mathcal{E}} - \theta^{\mathcal{D}}) = \frac{\partial}{\partial q}\theta^{\mathcal{E}} > 0$, follows from Proposition 4. $\square$

Proof of Corollary 7: Follows immediately from the discussion in the main text. $\square$

Proof of Corollary 8: Recall that the threshold $\theta^{\mathcal{E}}$ satisfies $N^{\mathcal{E}}(\theta^{\mathcal{E}}) = 0$, and the threshold $\theta^{\mathcal{L}}$ satisfies $N^{\mathcal{L}}(\theta^{\mathcal{L}}) = 0$. Recall that both $N^{\mathcal{E}}(\cdot)$ and $N^{\mathcal{L}}(\cdot)$ are monotonically decreasing in the threshold. To show that $\theta^{\mathcal{E}} < \theta^{\mathcal{L}}$ it is enough to show that $n(\beta) \equiv N^{\mathcal{E}}(\widehat{\theta}) - N^{\mathcal{L}}(\widehat{\theta}) < 0$. We note that $n(\beta)$ is decreasing in β and $\lim_{\beta\to 0} n(\beta) > 0$, while $\lim_{\beta\to\mu} n(\beta) < 0$. Hence, there exists $\beta^{delay} \in (0,\mu)$, such that $\theta^{\mathcal{E}} > \theta^{\mathcal{L}}$ if and only if $\beta < \beta^{delay}$. $\square$

Proof of Corollary 9: The comparison between $P_4(f,v)$ and $P_3(f)$ is straightforward. Here, we only compare $P_4(f,\emptyset)$ with $P_3(f)$. Recall that, $P_3(f) = qf + (1-q)\mu$ and $P_4(f,\emptyset) = \theta^{\mathcal{L}}$. Applying the Envelope Theorem, $\frac{d}{df}\theta^{\mathcal{L}} = \frac{1-p}{1-p+pG(\theta^{\mathcal{L}})} \cdot q + \frac{pG(\theta^{\mathcal{L}})}{1-p+pG(\theta^{\mathcal{L}})} \cdot \frac{q}{q+(1-q)G(\theta^{\mathcal{L}}} > q = \frac{d}{df}P_3(f)$. We note that $P_3(f = \overline{f}) = q\overline{f} + (1-q)\mu > \overline{f} = \theta^{\mathcal{L}}(f = \overline{f}) = P_4(f = \overline{f},\emptyset)$, because $\overline{f} < \mu$. Hence, we have $P_3(f) > P_4(f,\emptyset)$ for any $f \geq \overline{f}$. It remains to show that this inequality holds for $f < \overline{f}$. We note that

$$\begin{aligned}
\min_f P_4(f,\emptyset) &= P_3(f=0,\emptyset) \\
&= \frac{(1-p)(1-q)\mu + pG(\theta^{\mathcal{L}}(f=0))\left(1 - \frac{q}{q+(1-q)G(\theta^{\mathcal{L}}(f=0))}\right)\mathbb{E}[\theta|\theta \leq \theta^{\mathcal{L}}]}{1-p+pG(\theta^{\mathcal{L}}(f=0))} \\
&< \frac{(1-p)(1-q)\mu + pG(\theta^{\mathcal{L}}(f=0))\,(1-q)\,\mathbb{E}[\theta|\theta \leq \theta^{\mathcal{L}}]}{1-p+pG(\theta^{\mathcal{L}}(f=0))} \\
&< (1-q)\mu = P_3(f=0) = \min_f P_3(f).
\end{aligned}$$

Therefore, $P_4(f,\emptyset) < P_3(f)$ for any f. $\square$

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